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# The Use of Graded Originals in Plane Geometry

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THOUGH there is a definite trend toward the provision for individual differences within a class, in general, such provisions made in geometry have been based largely upon the idea of extra credit rather than upon differentiation in the materials used. Where such differentiation has been made, the materials provided for the gifted pupil are often more difficult than the rest, but this pupil must first do all, or at least part of the work done by the poorest students in the class. Thus the more able pupil is expected to do both more originals and harder originals than the rest of the class.

Teachers are aware of the fact that their best pupils often weary of problems which must be written in detail although they require little reflective thinking. They are also familiar with the discouraged pupil who is able to complete only a small portion of his assignment because he lacks both time and ability. These are the pupils for whom provision should be made.

In the University High School of the State University of Iowa, substitute assignments have been used for a good many years. Under this plan the pupil could substitute a harder original for any two or three easier originals or for all the rest of the lesson. On some days many pupils could work the harder originals, but on others they were so difficult that only one or two pupils were able to make the substitution. Sometimes the only way to get an A grade was to work the substitute assignment.

In this substitute work and in that done by other teachers, the difficulty of the originals employed has been chiefly determined by the opinion of the textbook author or the teacher.

Under the plan formerly used at the University High School for taking care of individual differences, it was assumed that it was desirable to use graded originals and to allow pupils to select their own originals. Out of the need to know more about the difficulty of originals and the efficacy of pupil selection has grown the study to be described. It is assumed that most readers will not be interested in the statistical details and other essential details relative to the research and so little has been included here. The complete report is in the library of the State University of Iowa under the title: *The Efficacy of Pupil Selection of Graded Originals in Plane Geometry.*

## PURPOSES

The objective of this study was to find out the efficacy of pupil selection of geometry originals when the work was organized to encourage intensive (depth of) thinking instead of extensive (breadth of) thinking. Throughout the experiment, pupils were encouraged to turn out quality

in their work rather than quantity. It was hoped that partial answers to such questions as the following could be obtained. Will the plan for pupil selection prove efficacious with all types of pupils—superior, average, and slow? How much will pupils make of the opportunity, and with what success, to do work as difficult as they are capable of doing, under the incentive of higher grades, less writing, and the possibility of spending less time?

What effects are there for the superior pupil who solves one difficult original instead of two or more simple ones? In doing so, does he have to think of fewer, the same number, or of more geometric concepts? Is his power increased, kept the same, or lessened by handling one difficult original instead of two or more easier ones? Is his speed in solving originals of average difficulty improved or diminished? Is his knowledge of the factual side of geometry affected by this substitution of depth for breadth?

What effects are produced in the average and the slow pupils by their being able to select originals? Does the pupil who works slowly become slower through receiving a passing grade for doing correctly a very small amount of work so that the time element is not a factor? Does he learn to solve more difficult originals? In short, is pupil selection of material sound methodology?

Pupil achievement in this experiment has been measured by tests, but it has also been measured by certain psychological effects connected with time, methods of approach, self-confidence, and indecision. The conclusions are based upon a detailed record of pupil behavior.

## The Experiment

Ten carefully selected, well-trained, and experienced teachers in nine different high schools took part in the experiment. The papers from 598 pupils were used in the final analysis. In all 18,276 papers were graded and tabulations made from them.

The schools participating were asked to try to have two groups of as nearly equal ability as possible. This point was not stressed, however, because at the end of the experiment the groups for comparison were to be selected from the experimental and control sections in each school by equating the means and standard deviations on two tests, *The Orleans Geometry Prognosis Test* and the *Otis Self Administering Tests of Mental Ability, Higher Examination.*

The teacher was asked to teach both sections under as nearly the same conditions as possible except for the differences in the use of the originals on twenty-eight special days devoted to the purposes of the experiment. Four periods of forty minutes each near the end of each unit but not necessarily on consecutive days, were spent on the graded originals by the pupils of both sections. Pupils were told that they *could* have their books open but *could not* receive help from the teacher or from other pupils. For the experimental group (Section *E*) there were twenty-four originals for each of the seven units. They were divided into three groups according to difficulty. The *A* group consisted of four difficult originals—more difficult than those commonly found in geometry textbooks for high schools—selected for the able pupils although others could try them. The *B* group, selected for the high average and above-average pupils, consisted of eight originals of at least medium difficulty. The *C* group was composed of twelve rather easy and easy originals for the low-average and poor students. The originals were labelled *A*, *B*, and *C* for Section *E*. The pupils were clearly told of the three levels of difficulty. They were required to choose the originals they wanted to work.

These twenty-four originals on each unit were divided into four sets, each containing one *A*, two *B*, and three *C* originals. Set Four was the easiest and was worked first. Set Three came next. Set One was the hardest and came last.

In the control group (Section *C*) twenty originals for each unit were worked during the twenty-eight special days. These were

the *C* and *B* originals as for Section *E*, in order from easiest to hardest, but with no comment of any kind to indicate the degree of difficulty. As in the case of the experimental section the originals for Section *C* were divided into four sets. The pupil worked from original number one of a set, through to number five, or as far as he could go in the forty-minute period. Aside from those important differences, and the grading of the work, the procedure for this section was the same as for the experimental group.

In the experimental group the pupils were given grades according to the number of points earned. If a pupil turned in the *A* originals in a set he could not turn in any other original. The only way to get an *A* grade for the day was to solve the *A* original. A grade of *B* was earned by working two *B*'s or one *B* and two *C*'s. A high *C* grade was earned by working three *C*'s and a low *C* grade by working two of them while a *D* grade was obtained by the correct solution of one of the *C* originals.

The grading system for the control group is a typical method of grading. Nothing was said to the pupils in this group concerning the method. They worked from the easiest original through to the hardest and the grade depended upon the number solved.

The originals used in this study were first constructed in connection with a preliminary study,[1] which was concerned only with obtaining a good set of graded originals and finding a suitable method of using them in a single public school. After a second year of use in the University High School the originals were regraded. For a check on the grading of difficulty of the originals, twenty well-trained, experienced high school teachers of geometry were asked to rate the relative difficulty of a sampling of the originals.

When the pupil results of the control groups were tabulated, it was found that, in general, they upheld the ratings given the originals.

Pupil results indicated that the teachers tended to underrate an original which is in the textbook as against one which is not a typical textbook original, and they did not make sufficient allowance for wording or lettering different from that commonly found in textbooks, although such a difference increases the difficulty considerably. Despite these facts, the data give adequate evidence that the collective opinion of a group of well-trained, experienced teachers is dependable in grading originals into three levels of difficulty, while the judgment of but one well-trained, experienced teacher is adequate for selecting very difficult and very easy originals.

Since the pupils in the experimental group were encouraged by the rewards to solve the hardest originals of which they were capable, and since if they solved the *A* original of a set they did not do further work on that set, it was to be expected that this group would solve fewer originals than the control group which worked as many originals as time would permit. During the twenty-eight special days the average pupil in the control group solved twice as many originals as the average pupil in the experimental group, in the upper quartile about two and one-third times as many and in the lowest quartile less than one and one-half times as many. When the number of originals solved by all of the pupils participating is compared with the number solved by the equated groups, it is found that the data for the larger groups substantiates the findings for the smaller equated groups.

### The Testing Program

Three final achievement tests were administered soon after the completion of the seven units of work or through the topic of circles. The first test consisted of the writing out in complete form, just as had been done on each of the twenty-eight special days, of one, two, or three originals. The first original represented an easy *C* ex-

[1] Payne, Lillian, *The Determination of a Graded Set of Originals for Plane Geometry and a Plan to Facilitate Its Use.* Unpublished Master's Thesis, State University of Iowa.

THE AVERAGE NUMBER OF ORIGINALS SOLVED PER PUPIL IN THE EXPERIMENTAL GROUP AND IN THE CONTROL GROUP IN THE TWENTY-EIGHT DAYS

Experimental Group

| 174 Pupils | Originals | | | Totals |
|---|---|---|---|---|
| | *A* | *B* | *C* | |
| Highest quartile on the two initial tests | 7.2 | 11.2 | 12.9 | 31.3 |
| Whole group | 3.1 | 6.6 | 14.7 | 24.4 |
| Lowest quartile on the two initial tests | .27 | 2.37 | 13.4 | 16.1 |

Control Group

| 163 Pupils | Originals | | Totals |
|---|---|---|---|
| | *B* | *C* | |
| Highest quartile on the two initial tests | 17.2 | 54.5 | 71.7 |
| Whole group | 9.4 | 38.4 | 47.8 |
| Lowest quartile on the two initial tests | 2.9 | 19.8 | 22.7 |

ample, the second an easy *B* example, and the third an easy *A* example. The second test consisted of fifty-five objective questions, Part 1 testing general information and Part 2 deeper thought processes. Test 3 was designed to test the pupil's ability to apply geometric reasoning to practical applications.

While the statistical assurance of the superiority of the experimental method is not all that could be desired, none of the evidence is contradictory, and in general it points toward the following conclusions. Which of the two methods is used for the pupils of low ability is probably immaterial as far as actual achievement is concerned. For the group as a whole, and particularly for the more able pupils, there is probably some, but not much, advantage in the procedure involving pupil selection. The trend toward the superiority of the methodology of the experimental method over the more conventional, while slight, is nevertheless present for all but the slow pupils. It at least suggests a very large and convincing superiority. The experimental method was used for only twenty-eight days which followed experiences in learning shared commonly by both groups and interspersed between identical methods of teaching on the other days which were far greater in number.

The plan for using graded originals with pupil selection and the efficacy of using them depend, however, upon more considerations than the results shown by achievement tests.

## THE AMOUNT OF TIME SPENT ON THE ORIGINALS

At the inception of this study it was thought that a record of the time spent by each pupil on each original might throw some light on such questions as these: Given a chance to select originals of *A*, *B*, or *C* difficulty, would pupils spend their time wisely in respect to their ability? With such opportunity might not less able pupils repeatedly attempt A originals to no avail? And might not lazy pupils be content to assure themselves of a passing grade and remain idle for the rest of the period? Would it take longer for an *A* pupil to work an *A* original than for a *B* pupil to work a *B* original or a *C* pupil, a *C* original?

Forty minutes were allowed on each of the twenty-eight days devoted to work on the originals. The average time spent per individual per day by the equated group in the experimental sections was 27.8 minutes and in the control sections 31.7 minutes. The correlation between results on the *Orleans Geometry Prognosis Test* and the total time spent on *A* originals by the experimental group was .640. Contrasted with that, the correlation between the same test and the total time spent on *C*

originals was −.604. In the control group, where everyone began on the *C*'s, the correlation with the total time spent on the *C* originals was −.333. The correlation with the time spent on the *B* originals was .235 in the experimental group and .253 in the control group. The *B* originals were attempted by nearly all the pupils in both groups. The correlations with the Otis test are not as high but show the same contrasts between *A*, *B*, and *C* originals.

It seems evident from these data that, in general, pupils chose to spend time on difficult or easy originals in accordance with their ability. The following table shows that the higher and lower quartiles of both groups spent about the same amount of time per day. However, the pupils of the highest quartile in the experimental group spent nearly twice as much time on *A* originals as on *C* originals, while the pupils of the lowest quartile in this group spent more than eight times as many minutes on *C* originals as on *A* originals.

All of the pupils in both groups attempted the *B* originals. Reasons for the much smaller amount of time spent on *B* than on *C* originals per day by the control group were these: There were three *C* and two *B* originals in each set; the *B* originals came last and were worked last, and the *B* originals were more difficult than the *C*'s. In the experimental group, all pupils of the upper quartile solved *B* originals, while about four-fifths of the lower quartile solved them.

All of the pupils of the lowest quartile solved one or more of the *C* originals and all but five of the upper quartile spent at least some time on *C* originals.

The few pupils in the lowest ability quartile who successfully completed *A* originals averaged five times as many minutes working on them as did the pupils in the highest ability quartile. Since only two of the *A* originals solved by any one in the lowest quartile were beyond Unit I, introductory material, the actual ratio of time for solution of *A* originals beyond Unit I is very much more than five to one. The lowest quartile spent about three times as much time in attempting *B* originals and three times as much time in attempting *C* originals for each correct solution as did the highest quartile.

THE AVERAGE NUMBER OF MINUTES SPENT ON A, B, AND C ORIGINALS PER PUPIL EACH DAY BY BOTH GROUPS

| | *A* | *B* | *C* | Total |
|---|---|---|---|---|
| **Experimental Group** | | | | |
| 174 Pupils | | | | |
| Highest ability quartile | 12.3 | 9.6 | 6.3 | 28.2 |
| Total equated groups | 7.4 | 9.0 | 11.4 | 27.8 |
| Lowest ability quartile | 2.2 | 7.1 | 18.0 | 27.3 |
| **Control Group** | | | | |
| 163 Pupils | | | | |
| Highest ability quartile | | 12.0 | 19.1 | 31.1 |
| Total equated groups | | 10.2 | 21.5 | 31.7 |
| Lowest ability quartile | | 8.4 | 23.2 | 31.6 |

All of the pupils of the highest quartile and the majority in the lowest quartile attempted *A* originals. The fact that so few pupils in the lowest quartile solved *A* originals (only eight pupils in this quartile solved *A* originals and six of these solved none beyond introductory Unit I) partly explains why the average time spent on these originals was not greater;—the pupils found them too difficult. The chief explanation is that the majority of the pupils merely looked over the *A* originals and, finding them too difficult, went on to easier ones.

In the control group, under the pressure of time to complete as many originals as possible, the highest quartile worked faster than did the highest quartile of the experimental group. Therefore, the average length of time for each correct solution is less for the upper quartile in the control

CONTRAST OF THE TIME SPENT ON SOLVING THE ORIGINALS BY THE HIGHEST AND LOWEST ABILITY QUARTILE IN BOTH GROUPS

| | | | |
|---|---|---|---|
| **Experimental Group** | | | |
| **Highest Quartile** | | | |
| Originals | *A* | *B* | *C* |
| Number solved | 295 | 453 | 527 |
| Average number of minutes to solve[1] | 47.9 | 31.1 | 13.7 |
| **Lowest Quartile** | | | |
| Number solved | 10 | 94 | 544 |
| Average number of minutes to solve | 251.5 | 86.5 | 38.1 |
| **Control Group** | | | |
| **Highest Quartile** | | | |
| Originals | | *B* | *C* |
| Number solved | | 704 | 2233 |
| Average number of minutes to solve | | 19.6 | 9.8 |
| **Lowest Quartile** | | | |
| Number solved | | 104 | 801 |
| Average number of minutes to solve | | 93.3 | 33.3 |

[1] This includes time spent on all attempts.

group. This more rapid work did not occur in the control group.

The average time required for the group as a whole to solve an original correctly was much less than the average time spent for each correct solution. The disparity here is largely accounted for by the fact that the *A* originals were made so difficult that even the best pupils were not expected to work them all while the rewards were so high that many pupils attempted them.

### TRENDS OF PUPIL SELECTION

The experimental procedure differed in these respects from ordinary classroom procedure: (1) It permitted the pupil to choose the level of difficulty at which he would work; (2) It offered big rewards in terms of a very high grade for a relatively small quantity of difficult work; and (3) It kept the pupil more aware of his immediate success or failure since it was impossible for him to secure a grade on any level of difficulty without mastery of the original. Such a procedure might well provide the answers to questions like the following:

Would all pupils attempt the more difficult originals, or would average and slower pupils be discouraged from the outset by their difficulty?

Would the pupil learn readily to appraise his ability for the task at hand or would indecision result in a great waste of time?

Would the more able pupil do his best to solve the more difficult originals without becoming over-confident?

Would pupils who found they could

THE AVERAGE NUMBER OF MINUTES SPENT ON EACH CORRECT ORIGINAL BY BOTH GROUPS

| | Experimental group 190 Pupils | | | Control group 180 Pupils | |
|---|---|---|---|---|---|
| Originals | *A* | *B* | *C* | *B* | *C* |
| Average number of minutes required to solve an original | 21.6 | 11.4 | 8.5 | 7.3 | 7.3 |
| Average number of minutes spent for each correct original | 65.7 | 37.6 | 22.1 | 29.5 | 15.2 |

solve only *C* originals while their classmates solved *A* and *B* originals develop as good understanding as in the ordinary routine and at least no greater feelings of discouragement and inferiority?

Would pupils be flexible in their selection, or would they choose one level of difficulty and stay by it, wisely or unwisely?

Would the plan of procedure be better suited to pupils of one sex than to those of the other?

Only nineteen pupils out of 174 made no attempt to work any *A*'s and everyone tried the *B*'s.

The four pupils who attempted no *C*'s were superior pupils able to succeed on the more difficult originals. Their number would bring down to three the number of pupils who did not succeed on *C* originals.

### The Relationship Between Initial Self-Appraisal and Success on Originals

Pupils tended to rate themselves as high or low in ability as evidenced by the fact that only two of them habitually began with the *B* originals. Over one-third of the pupils habitually began on the same one of the three levels. There were 18.6 per cent of the pupils who had sufficient confidence in their ability to start habitually with the *A* originals, *habitually* meaning that on 21 to 28 days out of the 28 special working days, these pupils began work on the *A* originals before attempting either *B*'s or *C*'s.

The average rating on the Orleans Test for the group habitually beginning with *A* originals was 120.2 whereas the average rating of the highest quartile in the experimental group was 128.7. This would indicate that it was not the most intelligent pupils alone who felt that they had the ability to solve the more difficult originals. In fact, of the 32 pupils who began with *A*'s habitually, six boys had ratings of 97 and below on the Orleans Test, while that of the lowest-rating girl was 101.

The average number of *A* originals solved by the group which habitually began with *A*'s was 8.7, while the average number solved by the highest quartile was 7.2. This does not mean, however, that the group habitually beginning with *A*'s made the highest work scores. Although these pupils gained over others of equal ability in the number of *A*'s solved, they solved fewer of the *B* and *C* originals. Thus, the group which habitually began with *A*'s had an average work score of 151.7, while the highest had a slightly higher work score of 157.0. Weights and checks were applied in determining the work scores.

But not only the work score of this group fell below the average of the highest quartile. Their average test records, also, were lower on each of the three tests than those of the highest quartile, although two of the boys with low Orleans Test Ratings made creditable records on the number of *A*'s solved and on the final tests.

These facts taken together indicate that in some instances pupils tended to rate their own ability too highly and that they persisted in trying the more difficult work when they would have done better to spend more time on *B* and *C* originals.

The Number of Pupils Who Did Not Solve and Did Not Attempt to Solve Originals of Each Level of Difficulty

| | 86 Boys | 88 Girls | Total |
|---|---|---|---|
| Did not succeed on *A* originals | 30 | 87 | 117 |
| Did not attempt *A* originals | 6 | 13 | 19 |
| Did not succeed on *B* originals | 4 | 11 | 15 |
| Did not attempt *B* originals | 0 | 0 | 0 |
| Did not succeed on *C* originals | 6 | 1 | 7 |
| Did not attempt *C* originals | 3 | 1 | 4 |

Only two pupils habitually began on *B* originals. This fact supports the statement that pupils tended to rate themselves as high or low in ability in beginning work.

Almost as many pupils—16.1 per cent—began habitually with *C* originals as began with the *A*'s. The average rating of these pupils on the Orleans Test was 66.4 as compared with an average rating of 58.4 for the lowest quartile in the experimental group. The highest rating was 96.

It was always pupils of low ability who habitually began on *C*'s. These pupils solved an average of 17.9 *C* originals, while the lowest quartile solved an average of but 13.4. Test results show that these pupils had a low average rating on all of the tests. Of the 28 pupils habitually beginning on *C*'s only 4 solved correctly any of the 3 originals on Test I, and these 4 solved only the first and easiest original. The teachers gave semester grades of *D* to each of the 28.

Thus, we see that it was not usually caution that prompted pupils to begin work at a low level. They did not habitually begin on *C* originals when they were capable of solving harder originals readily.

It is interesting to note the difference in the sexes in their initial choice of a working level. Boys were, without question, more daring. Of those pupils who habitually began on *A* originals, 23.3 per cent were boys, and only 13.6 per cent, girls. In view of the fact that this group included six boys with intelligence ratings of 97 or lower while the lowest ranking girl had a rating of 101, that the average intelligence ratings of this group were 113 for boys and 132.1 for girls, and that the work score average for boys was 135.7 as against 178.3 for girls, it is evident that boys had a much greater tendency than girls to reach beyond the level in which their intelligence ratings would have placed them. This greater daring is indicated also, though to a lesser degree, among those pupils who habitually began work on *C*'s, in the fact that the total average number of minutes spent daily on *A* originals by the boys was 2.0, and by the girls, 0.9; on *B* originals by the boys, 4.2, and by the girls, 3.4.

### The Tendency of Pupils to be Fixed in Their Habits of Choosing a Level of Difficulty

The following table shows that pupils were not, in general, set in their habits of approaching a task.

A comparison of the Orleans Test ratings and the work score ratings of those pupils who always began either on *A* or *B* originals shows correspondingly high scores. All but two had superior ratings on the initial tests and they had good scores. These pupils had high work scores. They were good students and knew that they were good.

Those pupils beginning always on *B* or *C* originals were slower and more cautious pupils, with an Orleans Test average of 65.8 and a work score of 36.7. These pupils solved no *A* originals.

From these data it becomes clear that in general the fixed habits of pupils in beginning work were suited to their ability and indicated the amount of their success. There were instances where daring or caution caused the pupil to begin work much above or below his level of accomplishment. It is also apparent from this table that most pupils were not set in their habits and were likely to begin work at different levels from day to day.

The Level of Difficulty at Which Pupils Always Began Working

| Number | | *A* *B* *C* | *A* or *B* | *A* or *C* | *B* or *C* | *A* or *B* or *C* |
|---|---|---|---|---|---|---|
| Boys | 86 | 1 0 0 | 11 | 9 | 7 | 58 |
| Girls | 88 | 0 0 1 | 14 | 2 | 18 | 53 |
| Total | 174 | 1 0 1 | 25 | 11 | 25 | 111 |

### Evidences of Beginning on a Higher or Lower Level of Difficulty Than the Later Level of Work for the Day

Very few pupils—only 13.3 per cent—stayed on one level of difficulty during any of the forty-minute periods given to work on the originals, or habitually worked either downhill or uphill in a single period. For example, a pupil might begin work on the *A* level, the *B* level, or the *C* level, but he might try, during the same period, some easier or harder originals than those on which he started.

There were eight boys who habitually stayed on one level of difficulty throughout the period. These boys ranked rather high on the initial tests, tended to work on the *A* originals, and did very well on the final tests. Only three girls habitually worked on the same level of difficulty throughout a period. These girls were low in ability, worked habitually on the *C* originals and did poorly on the final tests.

Two boys and seven girls habitually worked downhill. The averages for this group in intelligence, number of originals solved, and test scores were rather high. The three boys who habitually worked uphill scored low on the initial and final tests and solved no *A* originals.

For pupils to work habitually on the same level, or either uphill or downhill did not indicate surely whether they were likely to succeed or fail.

The typical person tended to rate himself as high or low in ability, probably too high as he worked downhill more than might be considered a healthy per cent of the time for the average pupil. He was not flighty in his work as he only wavered up and down on 2.8 days.

### Further Evidence on Sex Differences

The comparison of the average records of boys and girls gives support to the evidence on the greater daring of boys in working geometry originals. That this daring was not harmful is clearly indicated. Although the boys ranked lower than the girls on the initial tests and although their work scores were lower, they solved more *A* originals correctly and they ranked higher on all the final tests than the girls.

In the upper quartile the girls had higher ratings on the initial tests than the boys. They spent less time on *A* originals and solved fewer *A*'s than the boys, but they both spent more time on and solved more *B* and *C* originals, and they rated higher on all the final tests. In the lower quartile, boys and girls rated about the same on the initial tests. Boys spent more time on both *A* and *B* originals than girls, although the girls spent a little more time on *C*'s. The boys solved more *A*, *B*, and *C* originals than the girls, and they ranked higher on all the final tests.

Studies on sex differences in mathematics have shown that boys are superior to girls on geometry tests. It should be noted here that in this experiment, the boys ranked lower than the girls on the initial tests and higher than the girls on the final tests. There is rather more than a suggestion in these facts that the boys did not lose anything by their daring in selecting geometry originals and that they may actually have gained.

The Work Pattern of the Average Pupil

| | Average Boy | Average Girl | Average Pupil |
|---|---|---|---|
| Began work on A originals | 11.9 | 9.2 | 10.5 days |
| Began work on B originals | 6.3 | 8.0 | 7.2 days |
| Began work on C originals | 9.5 | 10.3 | 10.0 days |
| Stayed on same level during the forty-minute period | 11.5 | 10.9 | 11.2 days |
| Worked downhill during the forty-minute period | 10.4 | 11.5 | 11.0 days |

### Conclusions

1. The grading of the originals into levels of difficulty ranging from easy to hard was adequate, as shown by pupil work on

them. A check of the grading by experienced teachers of geometry indicated that such teachers can grade originals satisfactorily for practical use, especially as concerns the very easy and the very difficult.

2. Pupils in general learned to appraise their own ability. Because of the higher rewards for the more difficult originals, pupils often began work on a level of difficulty higher than their average accomplishment. However, most pupils tended to spend a large majority of their time on originals within their ability to solve.

3. The more able pupils attempted and solved more *A* originals than did less able pupils; on the other hand, less able pupils attempted and solved more *C* originals than did the more able pupils. The work of schools closely paralleled that of individuals—pupils in schools having a high average of ability attempted and solved more *A* originals; those in schools having a low average attempted and solved more *C* originals.

4. The average scores on each of the final tests were higher for the experimental group than for the control group. The critical ratio of the difference of the means on Test 2 for the two groups was not statistically significant. The fact that it was much higher for the highest ability quartile than for the lowest ability quartile suggests that the experimental method is better for the superior pupil than the method used in the control group, as far as achievement on an objective test is concerned. It suggests also that the experimental method does not help the pupil of low ability on an objective geometry test.

The results of Test 3 showed that the work with more difficult originals in the experimental group was probably a help to pupils, regardless of their ability, in preparing them to work on a geometry test involving practical applications of geometric principles.

5. Considering, along with test results, the number and types of originals solved, and the work habits of pupils, the following conclusions seems justified. The plan of pupil selection of graded originals in plane geometry was almost surely efficacious for pupils of high ability, and probably for pupils of average ability. Pupils of lowest ability were able to solve more originals when the element of pupil choice was not present (as in the control group), which may suggest that the procedure involving pupil choice discouraged them. There are no data to show that such pupils were helped by pupil choice.

6. The plan of pupil selection probably has some slight advantage for boys. (a) Boys were more daring than girls in attempting work on a high level, regardless of ability. (b) Boys solved more *A* originals than girls but fewer *B* and *C* originals, thereby sacrificing somewhat on work scores. (c) Boys placed lower than girls on the initial tests but somewhat higher on all the final tests. This suggests that their greater daring was not harmful and may have been beneficial.

## Recommendations

The data presented indicate that the procedure of pupil selection of graded originals might advantageously be used in the teaching of geometry, especially in classes ranking high in ability. In ordinary classes it might be used also if the welfare of the best pupils is considered to be of more importance than that of the slowest pupils. However, the welfare of the slowest pupils may be guarded by the use of graded originals in regular assignments not involving the time factor. This was not possible under the controlled conditions necessary to the experimental method under which all work was done within a given time and success or failure of the individual was better known to himself and to others than need be the case under regular classroom conditions.

In slow-moving classes there would be no advantage, and there might be possible harm, in permitting pupil choice with graded originals. For such classes the procedure followed with the control group, an ordinary class procedure, is probably better.

# The Story of the Parallelogram

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*Louisiana State University, University, Louisiana*

Recently I made a poll of a large number of students on the status of the rather uninteresting parallelogram. Discounting the defining statement that it has equal opposite sides, I could find no information at large other than "its diagonals bisect each other." Somehow this rather unpretentious figure has been laid aside to rust and I propose here to apply a bit of polish to see if its lustre cannot be brought forth in true brilliance. I think you will agree in the end that it does possess some memorable qualities which, although interesting, fascinating, and sometimes exciting, seem to come to light in a surprisingly simple way.

In the account to follow there will be much that can be transplanted to the high school classroom and yet some that might find receptive ears in college classes. In either or both cases, satisfaction may be added to pleasure only by actually making the models that are suggested by the pictures. Plenty of colored cardboard, a stout pair of scissors or photo trimmer, and an eyelet punch are necessary yet not difficult to supply.

## Parallelograms in Dissection

The art of dissection grows naturally out of the principle of "superposition" as practiced in the *Elements* of Euclid. There we are asked to establish equivalence of two plane polygons of equal area by imposing one upon another. If these polygons are not identical in size and shape, then it follows that we should be able to create partial polygons from each given figure in such a way that these partial polygons will be congruent in pairs. In this transformation from one given polygon to another by dissection the parallelogram plays a central role that is highly important.

Fig. 1. Let $P$, $Q$ be the midpoints of two sides of a given triangle $ABC$. If a cut is made along $PQ$ (which is parallel to and half the base) the top piece may be rotated about $P$ or $Q$ to produce either the left or right parallelogram.

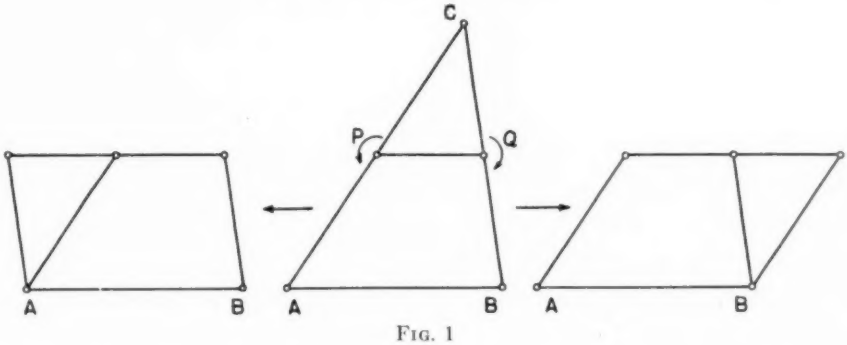


Fig. 1

Fig. 2. Given the parallelogram $ABCD$. From $A$ draw a line to an *arbitrary* point $X$ in $DC$. From $B$ draw an *arbitrary* line to meet $AX$ in $Y$. Let $u$ and $v$ be their lengths and $\phi$ their angle of inclination. Then $uv \cdot sin\phi$ is constant. If the fact is surprising, that it is obvious should be even more so: as $X$ travels along $DC$, triangle $AXB$ has a fixed base $AB$ and a constant altitude equal to that of the parallelogram. Thus its area is constant and equal to half the parallelogram:

$$uv \cdot sin\phi = ab \cdot sin\theta,$$

where $a$ and $b$ are sides of the given parallelogram and $\theta$ their included angle. Ac-

cordingly, if cuts be made along $AX$ and $BY$, the three pieces will form a new parallelogram as shown with sides $u$, $v$ and angle $\phi$.

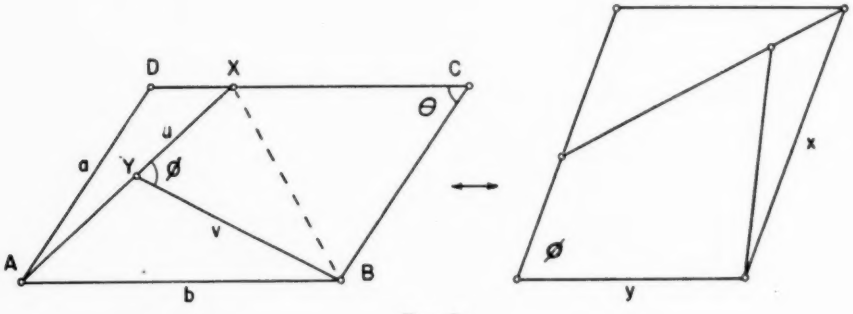


FIG. 2

As a corollary, note that if $X$ be coincident with $C$ and $Y$ be the midpoint of $AC$ then the area of the parallelogram is half the product of the diagonals and the sine of their included angle.

Draw the semicircle on $AG$ as diameter and extend $BF$ to meet it in $H$. Then $BH=\sqrt{(absin\theta)}$. Now locate $X$ so that $AX=BH$ and drop its perpendicular from $B$. Cuts along $AX$ and $BY$ give three pieces which may be assembled into the square $KM$.[1]

Fig. 4. The dissection shown here is the

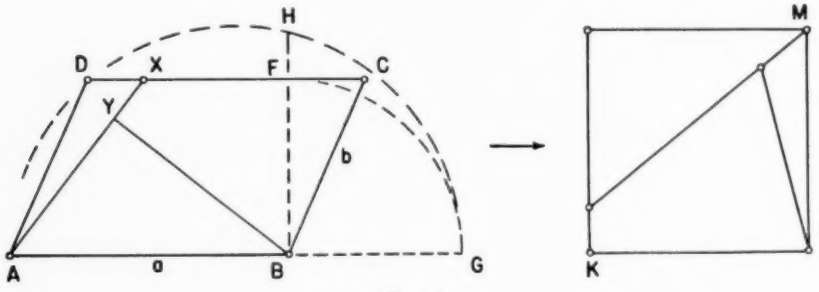


FIG. 3

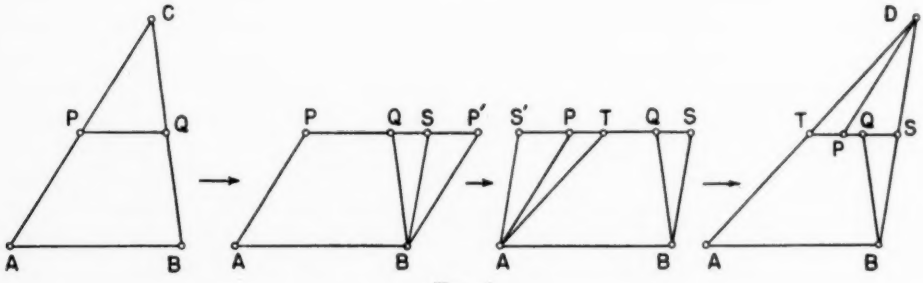


FIG. 4

Fig. 3. The preceding dissection may reduce a given parallelogram to a square of equal area. If the area of the parallelogram is $ab\cdot sin\theta$ then the side of the equivalent square is $\sqrt{(absin\theta)}$, the mean proportional between $a$ and $bsin\theta$. Construct as shown $BG=BF=bsin\theta$ on $AB$ extended. Draw the semicircle on $AG$ as diameter and extend $BF$ to meet it in $H$. Then $BH=\sqrt{(absin\theta)}$. Now locate $X$ so that $AX=BH$ and drop its perpendicular from $B$. Cuts along $AX$ and $BY$ give three pieces which may be assembled into the square $KM$.[1]

Fig. 4. The dissection shown here is the transformation from a given triangle $ABC$ to another, $ABD$, having the same base but with a specified angle $ABD=\theta$. As in Fig. 1, reduce $ABC$ to the parallelo-

[1] Certain restrictions will be noted here and elsewhere. Since for the most part they are obvious no special notes will be made.

gram $ABP'P$. Now cut along $BS$ so that angle $SBA = \theta$ and translate the triangular piece $BSP'$ to the left thus forming parallelogram $ABSS'$. Let $T$ be the midpoint of $SS'$. Cut along $AT$ and rotate the pieces $ATS'$ into their final position forming triangle $ABD$. If $\theta$ is too large or too small the dissection remains the same although more pieces are required.

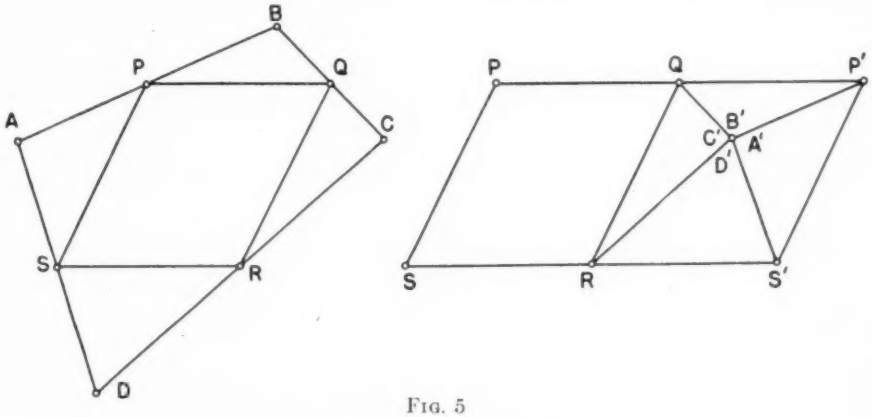


FIG. 5

This transformation is important in that it constitutes a method of reducing by dissection a polygon of $n$ sides to one of equal area having $(n\text{-}1)$ sides.

lar corner pieces is half that of the quadrilateral and therefore equal to the area of the parallelogram $PQRS$. These five pieces may be assembled to form the parallelogram $SS'P'P$, and it is at once evident that the area of any quadrilateral is half the product of the diagonals and the sine of their included angle.

### PARALLELOGRAMS IN MOTION

Now suppose we lift the parallelogram up out of its static existence by building it with jointed bars. This fresh kinematic interpretation enhances its character with many an added attraction.

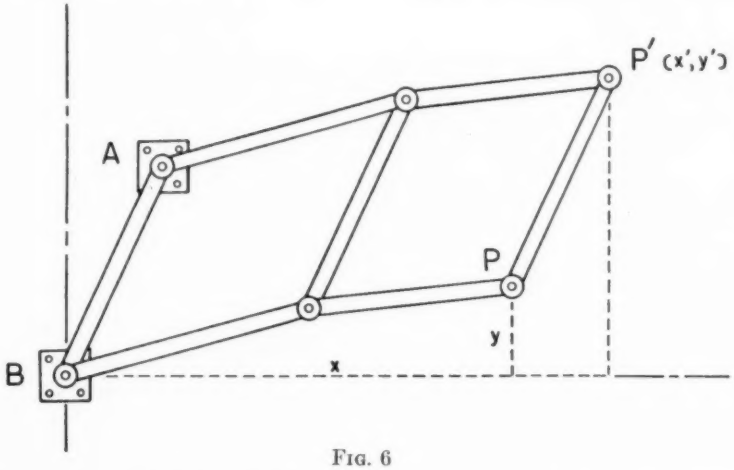


FIG. 6

Fig. 5. Given the quadrilateral $ABCD$. It is evident (see Fig. 1) that the midpoints $P$, $Q$, $R$, $S$, of its sides form the vertices of a parallelogram. The area of triangle $SAP$ is one-fourth that of $DAB$. Thus the sum of the areas of the triangular corner pieces is half that of the quadrilateral and therefore equal to the area of the parallelogram $PQRS$. These five pieces may be assembled to form the parallelogram $SS'P'P$, and it is at once evident that the area of any quadrilateral is half the product of the diagonals and the sine of their included angle.

Fig. 6. The points $A$, $B$ of the double linkage shown are fixed to the plane. Then within the limits of the mechanism, the point $P$ has two degrees of freedom. Let

$a$, $b$ be the horizontal and vertical projections of $AB$. Since the bar $PP'$ at all times remains parallel to $AB$,

$$x' = x + a, \quad y' = y + b,$$

the equations of translation which prove of use in analytic geometry.

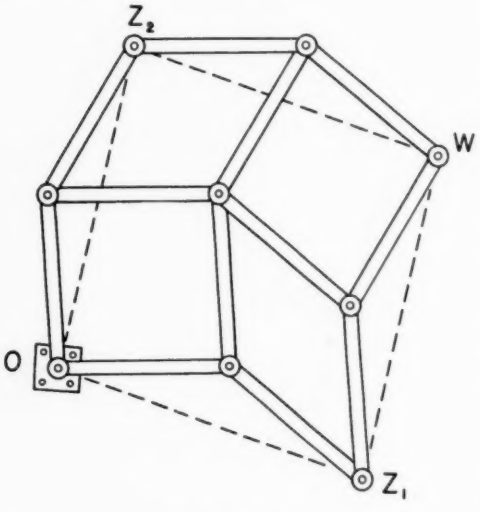


FIG. 7

Fig. 7. A combination of four parallelogram cells, restricted to a plane, will produce the complex transformation $W = Z_1 + Z_2$. It is obvious that the corners $O$, $Z_1$, $Z_2$, and $W$ form a parallelogram in any position of the linkage. If $Z_1$ be fixed, then $Z_2$ has still two degrees of freedom, but once $Z_1$ and $Z_2$ are both fixed, $W$ becomes fixed. The point $W$ may be thought of as determining the resultant of the two forces $OZ_1$ and $OZ_2$. Thus the linkage realizes vector addition.

Fig. 8. If the parallelogram be a rhombus, it may be collapsed so that all four bars fall coincident. Then it may be reopened in either of two ways as shown. Thus by fitting two such mechanisms to the top and bottom of a door, the door may be swung open in a clockwise or counter-clockwise direction, hinging at either the right or the left jam.

Fig. 9. The common pantograph is the parallelogram with two extended legs. Here two sides of the rhombus $RAQP'$ have been extended their own lengths to

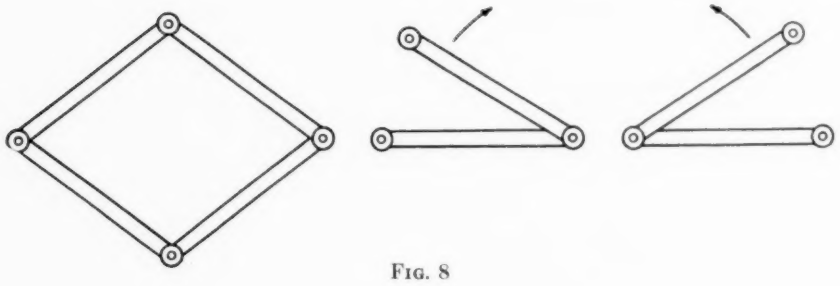

FIG. 8

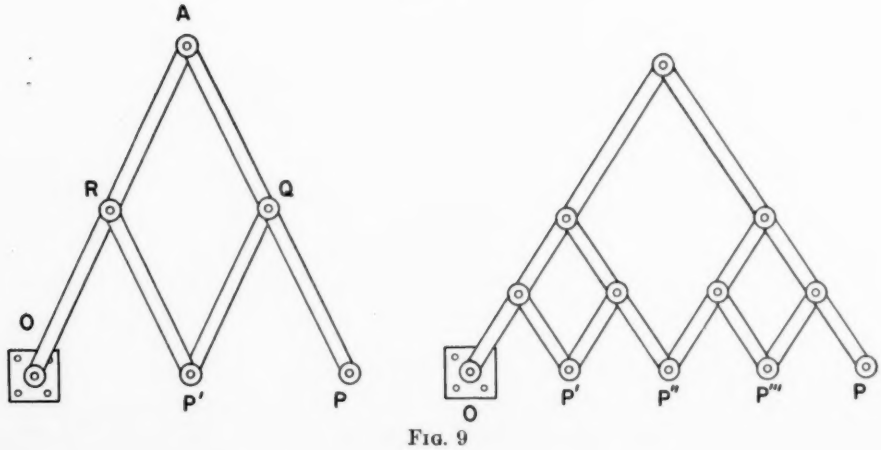


FIG. 9

$O$ and $P$. Obviously at all times, $OP' = RQ = PP' = (\frac{1}{2})(OP)$, with $O$, $P'$, and $P$ collinear. Thus if $O$ is fixed, $P'$ traces the same path as does $P$ except reduced by one-half. The extension of this to multiple tracers $P'$, $P''$, $P'''$, etc., as shown is immediate.

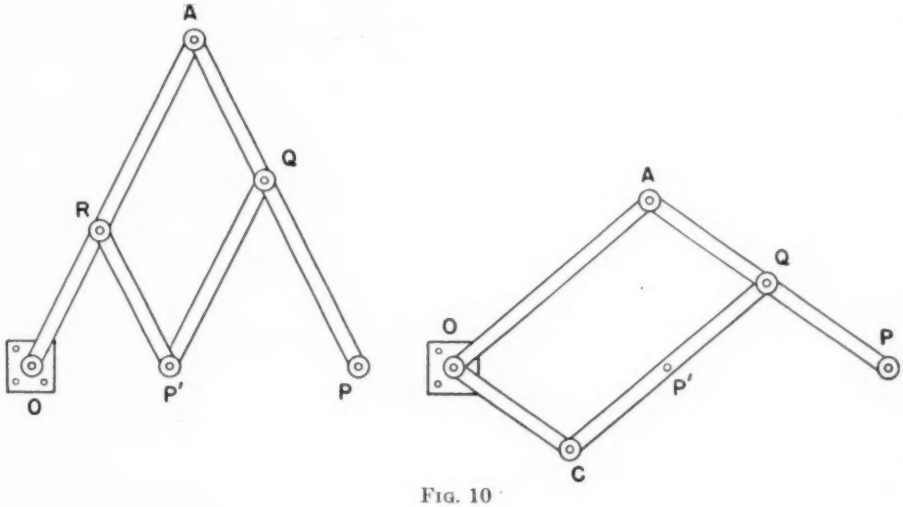


FIG. 10

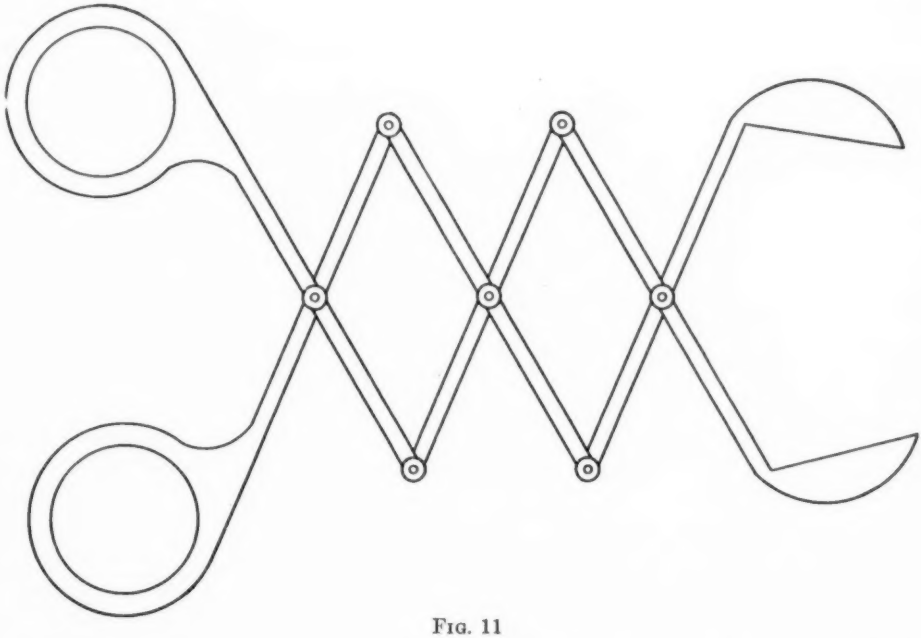

FIG. 11

Fig. 10. The pantograph need not be built upon a rhombus. If the general parallelogram $RAQP'$ has two of its legs extended to $O$ and $P$ such that $O$, $P'$, $P$ are collinear, then $P'$ copies $P$ with the reduction ratio of $OR/OA$. For, triangles $ORP'$ and $OAP$ are always similar and $OP'/OP = OR/OA = \text{constant}$.

Notice that the effect of the linkage is unchanged by lengthening the bar $QP'$ and moving $RP'$ parallel to itself so that $R$ becomes coincident with $O$. This is the pantograph shown in the right hand figure.

Fig. 11. The pantograph was put to good use years ago by grandfather when

he used it as lazy tongs to pull chestnuts out of the fire or to carry live coals from kitchen to living room.

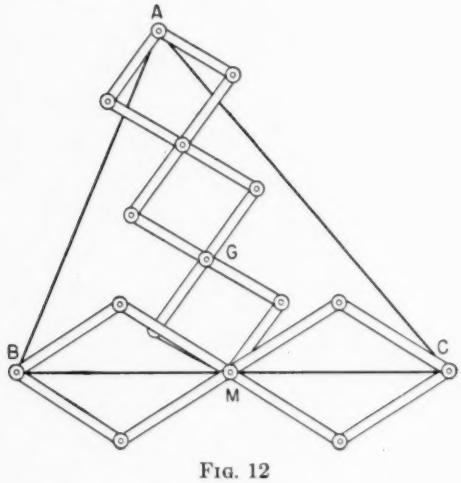


FIG. 12

Fig. 12. Five rhombuses are jointed together as shown. Obviously, $M$ is always the midpoint of $BC$ and $AM$ is a median of the variable triangle $ABC$. The point $G$ is two-thirds the distance from $A$ to $M$ and thus is invariably the centroid of $ABC$.

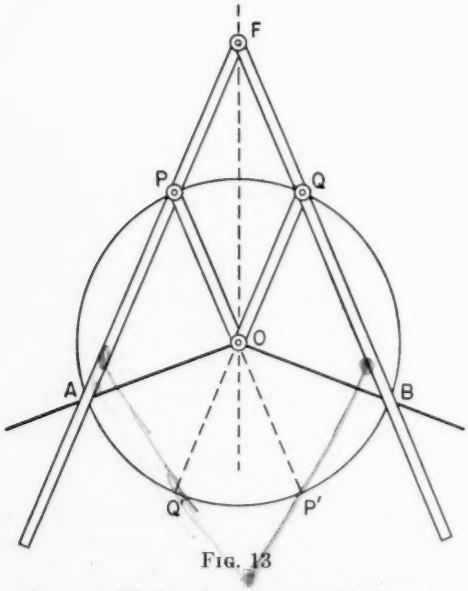


FIG. 13

Fig. 13. If the rhombus $OPFQ$ has two adjacent sides extended it is an immediate trisector of the general angle. Let the given angle be AOB and its bisecting line be $OF$. Draw the circle with center $O$ and radius $OP=OQ$ which meets the sides of the given angle in $A$ and $B$. Fix the point $O$ of the linkage at the vertex and move $F$ along the bisecting line until the extended legs pass through $A$ and $B$. Then angle $POQ=(1/3)(AOB)$. For parallel lines intercept equal arcs upon a circle.

Thus $\quad \overset{\frown}{PQ}=\overset{\frown}{AQ'}=\overset{\frown}{P'B}.$

But $\overset{\frown}{PQ}=\overset{\frown}{Q'P'}$, since these arcs are subtended by vertical angles. So

$$\overset{\frown}{AQ'}=\overset{\frown}{Q'P'}=\overset{\frown}{P'B} \text{ or}$$

$$\sphericalangle POQ=(1/3)(AOB).$$

Fig. 14. Two adjacent legs of a parallelogram may be replaced by similar tri-

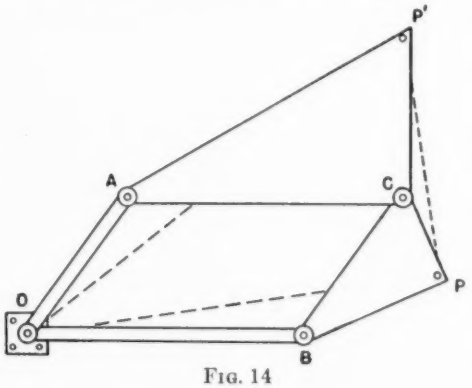


FIG. 14

angular plates to give the skew pantograph shown. If $O$ is fixed and $P$ traces a given curve, then $P'$ traces the same curve enlarged by the ratio of similarity of the triangles and turned through a constant angle. It is found convenient here to use the notation of complex numbers. Let the points $A$ and $B$ be represented by the variables $a$, $b$, respectively, and $P'$ and $P$ by $W:(x'+iy')$, $Z:(x+iy)$, respectively. Then since the triangular plates are positively similar:

$$W-a=Kb, \qquad a=K(Z-b),$$

where $K=\rho e^{i\alpha}$, $\alpha$ being the constant angle $P'AC=CBP$ and $\rho$ the constant ratio of the lengths $P'A/AC=CB/BP$. Accordingly,

$$W=K\cdot Z,$$

and the length $OP'$ is a constant multiple

of $OP$ while angle $P'OP$ remains constant and equal to $\alpha$.

Notice that the preceding equation establishes the similarity of triangles $P'OP$ and $P'AC$. Thus angles $OP'P$ and $OPP'$ remain constant and triangle $P'OP$ never changes shape.

A special case is of interest. If $\rho=1$, the two triangular plates are isosceles and

$$W=(e^{i\alpha})\cdot Z,$$

which, translated into the language of real variables, is

$$x'+iy'=(\cos\alpha+i\ \sin\alpha)(x+iy).$$

Equating reals and imaginaries, this produces

$$\begin{cases} x'=x\ \cos\alpha-y\ \sin\alpha \\ y'=x\ \sin\alpha+y\ \cos\alpha, \end{cases}$$

the equations of simple rotation. Thus by combining this mechanism with the one

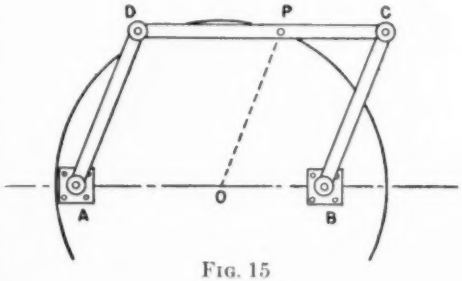


FIG. 15

shown in Fig. 6 we have a mechanical device for effecting a simultaneous translation and rotation that is so essential in the study of conics.

Fig. 15. The jointed parallelogram here has one side fixed to the plane. Select any point $P$ of the cross bar $CD$ and draw $PO$ parallel and equal to $AD$. It is clear that as the linkage moves with $AD$ and $BC$ remaining parallel the point $P$ describes a circle with $O$ as center.

Fig. 16. However, if opposite sides become crossed and no longer move parallel to each other, the mechanism assumes a very sophisticated character. Draw $PO$ parallel to $AC$ and $BD$ so that $A$, $O$, and $B$ are collinear, and let $AB=CD=2a$; $AD=BC=2b$; $(a>b)$. The reader may readily show that $(AC)(BD)=4(a^2-b^2)$. Select $O$ as the origin of a system of polar coordinates with $OB$ as axis. The coordinates of $P$ are $(r,\theta)$. Let $M$ be the midpoint of $AB$ and let $OM=c$. Let the distance $MT$ be $z$ where $T$ is the intersection of $AB$ and $CD$. Then

$$BD=2(BT)\cos\theta=2(a-z)\cos\theta$$

$$AC=2(AT)\cos\theta=2(a+z)\cos\theta,$$

and since their product is constant and equal to $4(a^2-b^2)$,

$$(a^2-z^2)\cos^2\theta=a^2-b^2.$$

But $r=2(c+z)\cos\theta$ and thus, eliminating $z$:

$$(r/2-c\cdot\cos\theta)^2=b^2-a^2\sin^2\theta,$$

the polar equation of the path of $P$. These curves, for various choices of $c$, are the inverses of the conic sections. As a special case, this last equation becomes, when $a=b\sqrt{2}$ and $c=o$ (so that $P$ is the mid-

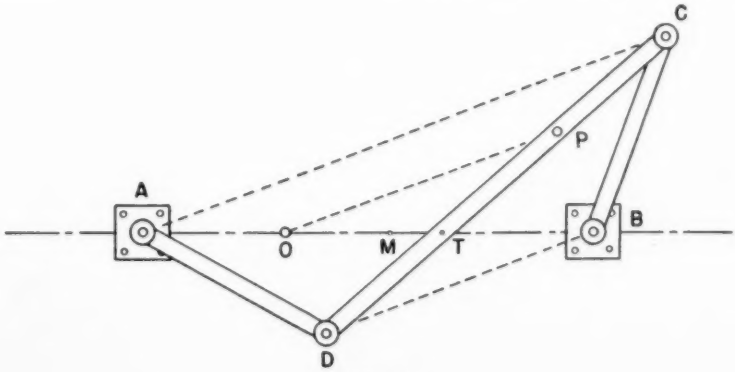


FIG. 16

point of $CD$):

$$r^2 = 2b^2 \cos 2\theta,$$

the equation of the familiar lemniscate, whose inverse is the rectangular hyperbola.

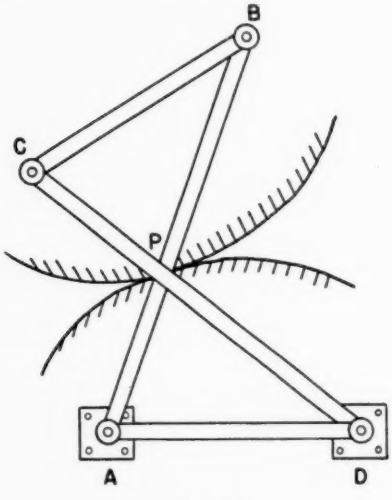


Fig. 17

Notice that in the crossed position of the parallelogram, the locus of $P$ is a quartic curve while that for the uncrossed case is of the second degree. We might, therefore, expect a locus of the sixth degree for

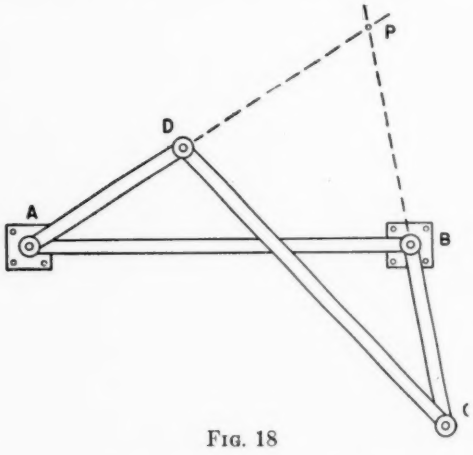


Fig. 18

a general three-bar linkage. This is, in fact, the case and the generated sextic will be found a study that is fascinating in its own right.

Fig. 17. By fastening one of the shorter bars to the plane we obtain the motion of an ellipse rolling upon another of the same size. Let $P$ be the intersection of the longer bars. Then since $AP = PC$, $DP = PB$,

$$AP + DP = AP + PB = 2a, \text{ a constant.}$$

The point $P$ thus describes an ellipse with $A$ and $D$ as foci. But, for the same reason, $P$ also lies on a moving ellipse with $B$ and $C$ as foci. These curves are tangent at $P$ and the rolling effect is apparent. Incidentally, if the bar $AD$ is released from the plane and $P$ forced to move along a line so that the directions of motion of the joints are always at right angles to the longer bars, the locus of a focus is the unduloid, a curve that plays a role in the theory of soap films.

Fig. 18. Now attach one of the longer bars to the plane and let $P$ be the point of intersection of the shorter bars extended. From the construction $DP = BP$. Thus

$$AP - BP = AD = 2b, \text{ a constant,}$$

and $P$ lies always on a hyperbola with $A$ and $B$ as foci. Again the effect is that of

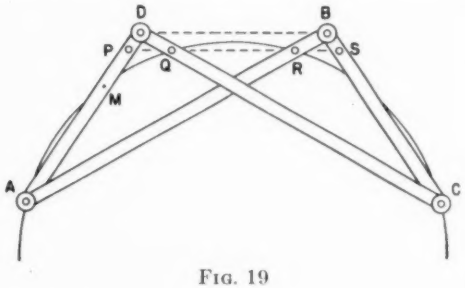


Fig. 19

rolling, here one hyperbola upon another. Notice that the directions of the asymptotes of the fixed hyperbola are defined by the poistions of the mechanism when $AD$ and $BC$ lie parallel.

Fig. 19. A most remarkable property of the crossed parallelogram is exhibited here. Let $PS$ be an arbitrarily chosen line parallel to $AC$ and $BD$ which cuts the bars in $P$, $Q$, $R$, $S$. Draw a circle passing through $A$, $Q$, $R$, and $C$ and let it cut $AD$ in $M$. Then, from the secant property of the circle,

$$(DM)(DA) = (DQ)(DC).$$

But $DA$, $DQ$, and $DC$ are themselves con-

stants and thus for all such circles $M$ is a fixed point of the bar $AD$. Accordingly,

$$(PQ)(PR) = (PM)(PA) = \text{constant},$$

and thus $Q$ and $R$ are inverse points with respect to a circle centered at $P$. By symmetry, $Q$, $R$, and $S$ also form an inverse set. The mechanism then is an inversor and performs the familiar transformation by reciprocal radii. It may, for instance, be attached to the linkage of Fig. 16 to produce the conic sections, the mechanism involving only seven bars.

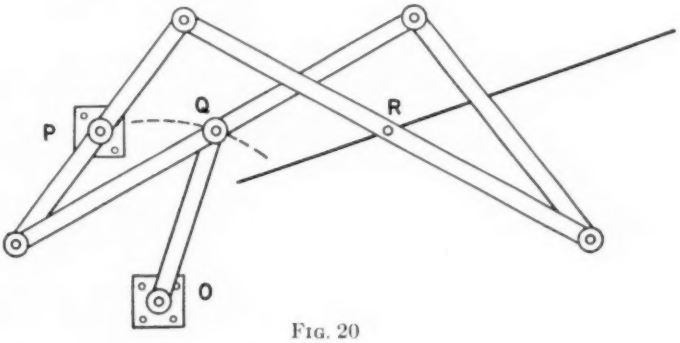


FIG. 20

Fig. 20. One of the well known principles of inversion is that any circle through the origin of inversion transforms into a straight line perpendicular to the diameter through the origin. Let $P$ be fixed at the pole of a system of polar coordinates and attach to $Q$ a fifth bar $OQ$ of any convenient length. Fasten $O$ to the plane so that $OP = OQ$. Let $OP$ be the polar axis and the coordinates of $Q$ and $R$ be $(\rho, \theta)$ and $(r, \theta)$, respectively. Then $Q$ travels on the circle whose equation is

$$\rho = 2(OP)\cos\,\theta.$$

Since, by the inversive property, $\rho \cdot r = k$, a constant, the locus of $R$ is

$$r \cdot \cos\,\theta = \text{constant},$$

a line perpendicular to the polar axis. This is the simplest known linkage for the generation of straight line motion.

Fig. 21. The union of two rhombuses enjoys the same inversive property that is inherent in the crossed parallelogram. It is clear that $P$, $Q$, $R$, and $S$ lie always upon the line bisecting angle $APB$. The points $Q$ and $R$ travel upon a circle with center at $A$. By the secant property,

$$(PQ)(PR) = (PA+b)(PA-b) = a^2 - b^2,$$

where $PA = a$, $QA = b$. Thus this double cell may be used for inversion in place of the crossed parallelogram. Either of the pairs of bars $(SA, SB)$ or $(RA, RB)$ may

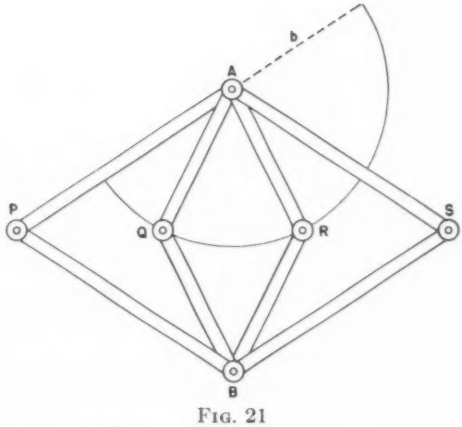


FIG. 21

be discarded and the remaining three joints which lie collinear will form an inversive set.

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# Mathematics for the Modern Junior High School*

*By* Earl Murray and Robert W. Ritchie
*Santa Barbara, California*

A unique feature of this session was the absence of any prepared paper or set speech. Nevertheless, the session was planned carefully and laboriously. Each member of the panel was secured under the arrangement that the session was to be a frank and open discussion of junior high school problems. The chairman prepared mimeographed sheets with suggestions for discussion which were mailed to each panel member about six weeks before the date of the session. Each member responded to these suggestions in writing, sending the material to the chairman, who mimeographed it and returned a copy to each panel member. Because of this, each of us felt acquainted with the others and their viewpoints on the problems, even though we had never seen one another. We did not meet until the hour of the session. The suggested material sent out by the chairman will appear at the first of the discussion under each main topic, rather than appearing here as one document.

*Problem I. Where shall we begin mathematically in the seventh grade?*

The suggestions of the mimeographed copy were: "In line with modern tendencies in the elementary school to promote to junior high school on age basis and other bases than achievement of skill, may we assume that there is no positive beginning point for seventh grade mathematics other than a fair knowledge of the four fundamental processes?

"Does this seem to be true in your school? What have you done or are doing to meet this changing condition?

"Do you find that you must begin with seventh graders where they are, rather than assuming that they have at their disposal a set of memorized mathematical skills? If in your system you can draw a line or state the mathematics situation in which you find your incoming students, please do so.

"What is the basis for division of mathematics classes in the seventh grade?"

This problem brought out some interesting points which led to an equally interesting conclusion. It became evident that throughout the country in general, old standards of achievement had been broken down. The junior high school teacher, instead of receiving groups of entering seventh grade pupils equipped with the set of memorized mathematics concepts once considered necessary for "passing" the sixth grade, is now faced by entering classes which fall about a year short of entering classes of bygone days. When this tendency became apparent, a quite common complaint of the junior high school teacher was "the elementary teaching isn't what it used to be." The sober reflection and common sense of many junior high school mathematics teachers quickly refuted this idea; but still left many puzzled. Later, as the situation became more evident, its cause became more obvious. As brought out in our discussion (mainly by Dr. Lefever of the University of Southern California and Mr. Lindsay of the California State Department of Education) there are a number of contributing factors, some of which are:

1. A general realization by elementary teachers of the validity of the contention, supported by many studies, that postponement of formal arithmetic in the earlier grades is beneficial. It was pointed out by Dr. Lefever that the work of the *Committee of Seven* emphasizes the importance of mental maturity as a factor in readiness for learning arithmetic. Evidence gained from studies seems to indicate that a great

* A report of a session of the National Council of Teachers of Mathematics on the subject at Berkeley, California, July 5, 1939.

deal of valuable teacher time has been consumed in attempting to teach arithmetical concepts to children of the lower elementary grades before they are capable of absorbing them. Children who have had little or no formal arithmetic in earlier years soon "catch up" and display an adeptness equal to that of pupils of their own chronological age who have had the benefit of formal arithmetic training in each grade. Carrying this reasoning to its logical conclusion, many systems have decided that sixth grade pupils are not mentally ready for a systematic study of decimals, percentage and fractions and their inter-relatedness, and consequently have moved the systematic and formal study of these concepts up into the seventh grade.

2. No failure program. The adoption of the "no failure program," that is, promotion upon the basis of chronological age for socially desirable reasons, rather than upon the basis of achievement, has had at least one important effect. It lowered the average chronological age and consequently the mental maturity of the entering seventh grade student. Again referring to Dr. Lefever: "In several school systems in Southern California either chronological age grouping or the no-failure program have resulted in a change in average chronological age for entering seventh grade pupils from twelve years and eight months to an even twelve years. This decrease of some eight months in chronological age means a definitely lower level of maturity for seventh and eighth grade pupils. In one medium-sized school system over six hundred pupils in the seventh and eighth grades were checked; a comparison was made between the arithmetic skill possessed by the seventh and eighth grade pupils in 1932 under a traditional promotion class with the arithmetic computation skill of the seventh and eighth grade children in 1935 after a definite no-failure program had been established. The mean grade placement for 1932 at the beginning of the seventh grade was 7.5, in 1935 the grade placement was 6.0. This decrease in arithmetic grade placement is due in large measure to the change in average chronological age because on both the Progressive Achievement Test Manual and the new Stanford Achievement Manual the grade placement which corresponds to twelve years and zero months chronological age is 6.2 not 7.0. In other words these children are doing as well as can be expected for their chronological age, but they are not living up to traditional standards for a grade placement which was formerly rated as seventh grade. I do not mean to suggest that a no-failure program is necessarily bad, but I do believe that such a program should be accompanied by a very careful adjustment of the arithmetic curriculum to the changes in pupil maturity."

Although this study referred to a particular locality, it confirmed opinions unanimously expressed by members of the panel in the questionnaire sent out by the chairman previous to the meeting. Each member, in one form or another, expressed the idea that no fixed standard of mathematics skills could be expected of entering seventh grade students.

3. A third factor which has had somewhat similar effects is the adoption of an annual promotion program in many schools. Elimination of mid-year registration and mid-year promotion make it possible for a child to enter junior high school at the age of 11 years, 6 months.

The conclusion drawn by members of the panel was that present conditions should be recognized and proper adjustments made. The situation seems to call for a careful re-analysis of the elementary program and a consequent adjustment of the seventh grade curriculum.

*Problem II. What is the place and function of drill in the learning process?*

The suggestions of the mimeographed copy were: "Are we trying to teach, in our mathematics classes, skills and knowledge of processes as the outgrowth of

1. Maturation growth of child,

2. Comprehension of processes involved, and
3. Meaningful use in student problems?

When and what part does drill play in this learning process? Give some illustrations of where drill has been used in your classes as an outgrowth of student demand because of properly motivated comprehension and use of a process. Give illustrations of (busy work) meaningless drill—and result on child."

The discussion on drill served to emphasize its importance and its dangers. Dr. Christofferson cautioned against the use of drill without first arousing in the pupil a consciousness of a lack of proficiency that should be remedied. If the child is made to feel that he needs practice in any particular process, the deadly monotony which is associated with meaningless drill is avoided. Dr. Christofferson was seconded by Dr. Lefever, who pointed out that drill should follow functional problems filled with socially useful content, rather than precede these problems. Mr. Patton of Fresno explained an unusual and interesting technique used in his entering seventh grade classes as a review and prelude to practice in fractions. He uses a motion picture film, made by himself, which takes the student through a review of addition of fractions and arouses a desire for greater proficiency.

Miss Wilson of San Francisco pointed out that drill, after need had been aroused, should be continuous until accuracy and speed result.

*Problem III. Is there an established mathematical standard for passing the eighth grade?*

The suggestions of the mimeographed copy were: "In your school system, are the students supposed to attain a certain standard of achievement in the manipulative skills in order to pass the eighth grade?

"Is this standard of achievement quite rigid? If not, just how flexible is it?

"Note: The answer to this should bear upon your attitude toward ninth grade mathematics."

None of the members of the panel reported a fixed standard required to pass the eighth grade. Each member seemed to feel that a policy which demands an attainment of fixed standards by all pupils was indefensible, and that the present practice of attempting to extend each child to the limit of his ability was not only desirable, but the only practical one.

Several of the schools represented used the results of eighth grade work as a basis for counseling the type of mathematics to be pursued in the ninth grade. Miss King of Oakland is using this method quite satisfactorily.

*Problem IV. How is mathematics and the core inter-related?*

The suggestions of the mimeographed copy were:

"Do you have a 'core' course in your Junior High School? If so, how many hours daily are devoted to it? What general theme or subject matter is it built around? How much mathematics is in it? How much time does a mathematics teacher spend in helping core teachers with the mathematics skills? Are any of the core teachers, or team of core teachers, specialists (or adequate) in mathematics? Have any of them taught a mathematics class, or are now teaching a mathematics class as well as a core class?

"How much mathematics and what mathematics can be taught in a core course and what should be left for a specialized mathematics class? For instance, it is quite probable that certain basic mathematics experiences could be taught more functionally in a core class. The understanding of certain instruments, graphs, maps, charts, which were once called mathematics are now considered to be problems of reading and social studies. Do materials now taught in a junior high school class need to be revised? Are we constantly revising these materials? How

can certain integrative mathematics experiences and materials be supplied to the core teacher and classes? Suppose there is a need for mathematics knowledge in a core experience which is beyond the logical mathematics development of the group; interest has been aroused, learning readiness is apparent, what shall we do about it?

"Should we strongly suggest units for the core that have a great amount of functional mathematics in them, such as units on business, home, consumer problems, taxation, insurance, etc.?

"Can we keep on teaching mathematics in a logical sequential development and keep up the interest? What is the breadth of function of the mathematics teacher in his two fold duty of

1. Contribution to core areas (material for classes and helping teachers) and
2. Developing and satisfying those with special interest in mathematics?"

Santa Barbara being the only school represented which had a core course, Mr. Ritchie was asked to speak on this problem. His remarks, in effect, were as follows: We do have core courses in our school. In the seventh grade three hours are devoted to the core, while in the eighth and ninth grade but two hours are thus used. The core takes the place of subjects formerly known as English, social studies and science, but should not be considered as a combination or "throwing together" of an English, a social studies and a science class, since the work in each core is built around some integrative theme. The amount of mathematics taught in the core varies, since some teachers, especially those who have taught mathematics, are quicker to see the functional mathematics possibilities than others. Mathematics teachers do not devote any set time to assisting core teachers but there is by and large, a very healthy spirit of co-operation. The mathematics teachers, on one hand, point out mathematics possibilities in the core, while the core teacher, on the other hand, requests the mathematics teachers to assist by clarifying mathematical concepts or processes that may be needed in the core. On close examination it will be found that most core units are rich in functional mathematics possibilities, so that while it may be advisable to occasionally suggest a unit which has a great amount of functional mathematics, as a general rule it would be better to allow the core teacher to make his own decisions. After all, the core teacher is not expected to teach the pupil new mathematics, but to encourage the use of whatever mathematics knowledge the pupil possesses in any functional situation which may be encountered in the core. The introduction of the core has not lessened the amount of time set apart for mathematics, since each pupil must spend a period a day in a mathematics class.

As further clarification of the Santa Barbara viewpoint we quote from the *Report of Arithmetic Committee*, Bulletin II-b, Oct., 1938, Santa Barbara City Schools.

"*Arithmetic and Mathematics for Junior High School:* In the junior high school the committee feels that the core course teachers should be helped to see the contribution that number can make to a clear understanding and enrichment of those concepts and principles which form the content of the core course. Many social concepts cannot be adequately understood unless clarified by the use of number. Such terms as 'Standardization,' 'Mass Production,' 'Economics,' 'Standard of Living,' etc. are practically meaningless unless number is introduced into the explanation. Effective comparison is likewise impossible unless mathematics is used. For example, in studying the shift from the use of coal to oil; percentage is used to clarify the problem of why such a shift has taken place, a much more effective procedure than that of developing the concept of percentage and searching for problems that illustrate it. We are not asking that core teachers stress the development of ratio, percentage, decimals,

etc., and look for illustrations of these in their units, but rather that they be aware of the contributions of these number skills and use them to facilitate the understanding and clarification of the problem under consideration. Members of this committee will work individually or with groups of junior high school core course teachers in helping them to become sensitive to the opportunities for arithmetic in their core course previews.

"It is understood that in the core courses the use of number is primarily to clarify and enrich the concepts basic to the content under consideration.

"If, in the utilization of these mathematics skills in the core courses, the teacher should find instances of skills which need building up; it would be his duty to report these deficiencies to the mathematics teacher with the recommendation that they be remedied in the regular arithmetic class. In such event the mathematics teacher irrespective of his current plan will immediately give time to the development of such needed skills. It is apparent, therefore, that the core course teachers do not carry the responsibility of developing or systematizing new skills and processes. Rather, they ask the co-operation of the mathematics teachers when such articulation will facilitate the core course.

"In grades VII, VIII, and IX, there will be special attention given to arithmetic, and all of the pupils will be enrolled in these courses. In the seventh and eighth grade arithmetic, the teachers will be responsible for systematically developing the concepts of, and skills in fractions, decimals, percentages—all of which have to do with proportion. Again, it must be remembered that the enriched arithmetic experiences of the elementary grades will already have developed considerable insight into these processes, and, in many cases, mastery of the simpler fractions, decimals (particularly as pertaining to money), and some grasp of percentage. In the formal arithmetic work of grades VII, VIII, and IX, the instruction will continue these beginnings of the elementary school and systematically fix them. Again, this postponement of processes that have formerly been attempted in the elementary years is made with the thought that the pupils will have matured sufficiently, at these later junior high school years, to master much more adequately these important arithmetical concepts and skills.

"Again, in the junior high school the arithmetic committee will discuss with the shop teachers and the household arts teachers the manner in which they use functional arithmetic, to the end that the successful experiences of these junior high school departments can be made available to the general faculty. It is recognized that these special departments have, for some time, successfully developed arithmetic in connection with their shop and household arts projects.

"*In Junior High School Mathematics:* All pupils shall be given as rich and as comprehensive a development of advanced arithmetic and elementary mathematics as the maturity and interest of the junior high school pupils can absorb.

"It is supposed that the mathematics teachers will subject their classes to a wide variety of mathematics always in a functional and meaningful way. A record of pupil responses to each type of mathematics presented will be kept for a purpose of arriving, eventually, at a curriculum for the grades, seven, eight, and nine.

"Concurrently throughout the three years attention will be given to the development of skill in the use of fractions, decimals, and percentage. The work in mathematics in the senior high school presupposes a mastery of these skills in the junior high school—a mastery which the functional experiences in the core courses are not adequate to develop."

In the discussion which followed Mr. Ritchie's presentation, the chairman was asked to explain the relationship between mathematics and the core in the senior high school. Mr. Murray stated that there are no true core courses in Santa Barbara

High School at this time, but there are a few experimental courses preparatory to the adoption of a core. Being willing to express himself concerning the issue, he proceeded, in effect, as follows:

Mathematics teachers should support the core idea rather than oppose or be indifferent to it, because it is our best opportunity to get needed materials taught to all students. At the present time less than 50% of our high school students take any mathematics course during their high school career. The courses which this minority study are designed for college preparation, are generally pre-vocational and technical, and allow little time for cultural mathematics or "units about mathematics." The present tendency is certainly not toward any more required mathematics, but rather toward a lessening of the requirement, so that our present 50% may shrink considerably. We are very unhappy about this situation because we are aware of the usefulness of our subject. The solution does not lie, however, in berating the curriculum director for not seeing the need. He is continually beset by pressure groups demanding more courses. A study of this situation reveals that should time be given for all "subjects" or "topics" to be taught to all students that are requested by various groups, we would be forced to have a five year high school, and then students would have little time to spend on their special interest field. The very best we can hope for in the way of required time is a mathematics class designed for some remedial work in the skills of arithmetic.

There is a growing demand for the study of such topics as safety, consumer education, and family relations in the curriculum as well as the ever present pertinent and persistent problems of our social and economic life. At the present time certain phases of these topics are quite adequately discussed in the shop, homemaking, science, mathematics, English, social studies, physical education, art, music, and commercial classes. The difficulty is that a student not being exposed to all of these courses, is apt to get a partial view, or an overlapping of materials, rather than the wholeness of the problem. It is an education by parts and frequently unrelated and separated by large periods of time. The modern way would seem to be to study these problems by wholes, bringing in the contributions of the various school specialists as they are needed to solve the problem under discussion. In this way our learning will become more functional. It will then be our duty as mathematics teachers to work with core teachers in helping them to use the functional mathematics which we believe is needed for the intelligent solution of these life problems.

Mr. Murray proceeded to mention a few of the contributions which he believes we can make to the core program.

1. A better knowledge of installment buying, as a contribution to consumer education. Recently much more readable and understandable material has appeared on this subject, and it would seem that we ought not to allow students to know nothing about this subject just because we fear that a core teacher might not teach everything about it.

2. A better understanding of gambling, horse racing, slot machines, and other games of chance, as a contribution to broader social problems. The general public and youth in particular are wary of these things, but a mathematical knowledge and analysis is much better and more effective in building good citizenship than is a superstitious fear.

3. Accurate and reliable collection, statement and interpretation of statistics for the understanding of economic issues. We need the use of and at least partial understanding of food indices, standard of living levels, home budgets, rates of taxation, social security schemes, old age pensions, and the ability to use statistics to predict trends.[1]

[1] Rosander, A. C. "Mathematical Analysis in the Social Studies." THE MATHEMATICS TEACHER, Vol. XXIX, No. 6, Oct. 1936.

4. A better understanding of insurance, investments, savings, and annuities, as a contribution to business, social, and economic problems. It is admitted that much of these subjects is beyond high school age and the simple mathematics mastered by every high school youth, but many formulas and fundamental rules could be used, even though with but partial understanding, in much the same manner that Junior High School youth studies and uses mensuration of geometric figures long before he is able to understand the proof of those formulas.

5. A better and more useful interpretation of data. This is a process of thinking and relates to all fields of subject matter. We need to learn to distinguish between relevant and irrelevant data, evaluate dependability of data and adequacy of method of presentation, formulate conclusions in light of the data, and refrain from generalizations when data is insufficient.[2]

6. A better understanding of logical thinking and the nature of proof. This is certainly desirable in all phases of living. It includes the importance of definitions and assumptions in non-mathematical situations such as government, advertising, ideals of conduct in school and community, and attitude toward law observance and enforcement. This is basic to human happiness for integration is the essential unity of assumptions of conduct with actions of behavior. We need a thorough understanding of induction, deduction, generalization, analysis, and the role of authority in the reasoning process.[3]

The things just mentioned have all been taught in mathematics classes where only a small percentage of our students are exposed to them. They are really not mathematics. They are essentially *life experience* units, which need all departments of learning to solve. It would seem that our most probable chance of getting every student exposed to these essentials is to support a core course centered around the basic needs of the individual and then do our part in the solution of the problems.[4]

*Problem V. What shall be our mathematics program for the ninth grade?*

The suggestions of the mimeograph copy were: "What mathematics is required of all students in your school? Is there a definite three year sequential development, or does the course stop with the eighth grade, and then something new developed for the ninth grade? Is some form of mathematics required in the ninth grade? If not, how many students (per cent) do enroll in ninth grade? What is your school set up in ninth grade? Do you think you should have a sequential three year program with some adjustment for individual and group differences? Do you think students should branch out into special interest fields in ninth grade such as shop mathematics, business, general mathematics, algebra? Do you think algebra should be taught in ninth grade? If algebra is taught, should we have two types of algebra—one for those who plan to use it in a professional field and one for those who take it only because it is demanded for college entrance?

"Should ninth grade be the beginning of specialization in mathematics or should the ninth grade mathematics course be of general educational value to all?

"Is the student who takes algebra missing other mathematics experiences he ought to have?

"What should be the basis for division in the ninth grade?

"How are these things done in your school?

[2] *Progressive Education Association Evaluation in the Eight Year Study.* Bulletin No. 3, Oct. 1935. Bulletin on Interpretation of Data.

[3] The National Council of Teachers of Mathematics. Thirteenth Yearbook, *The Nature of Proof.*

[4] *Mathematics in General Education.* Commission on the Secondary School Curriculum of the Progressive Education Association. Revision of June 1938.

Materials Prepared by Participants in the Mathematics Group of the Progressive Education Association Summer Workshop at Sarah Lawrence College, Bronxville, N.Y. July 2–Aug. 13, 1937.

"Are you planning any change or revision?

"If there should be one course for all in the ninth grade what should be the content? Combination of shop mathematics, business, simple and functional algebra, intuitional geometry, etc.? Upon what basic philosophy would you base your choice of curriculum materials?"

At the present time nearly all, perhaps 98% of our ninth graders are enrolled in a mathematics class of some kind. A few schools require mathematics of all students, but in a far greater number, the students are guided into it. The most common courses are algebra, junior business training, general mathematics, or shop mathematics. Some members of the panel thought that this arrangement is satisfactory, that the student should be guided intelligently into his interest field and that many should be allowed the opportunity to choose a year course in algebra at this time. Other members supported the postponement of formal algebra and the formulation of a ninth grade course based upon the "functional needs" of the students.

Dr. Christofferson made the following statement, although time forbade any enlargement upon it: "I'd like to support briefly the contention that mathematics through the ninth grade should be so modified as to be of general value to every boy and girl. No matter what the occupation or profession, everyone has need for quantitative thinking of some kind and even logical reasoning of the type illustrated in tenth grade geometry. Specialization into shop mathematics, business mathematics, etc., should follow a general course packed full of illustrations of the use of mathematics in shops and business."[5]

Dr. Lefever had previously pointed out that the chronological age grouping and no-failure program had resulted in a reduction in average chronological age of eight months. This factor, coupled with various studies which have been made in the elementary schools showing rather definite gains from the postponement of formal arithmetic, would lead to the conclusion that more advanced placement of arithmetic topics would be advantageous.

Dr. Lefever said: "It would seem to be logical then to provide for maintenance activities in the later years of the junior high school and on into the senior high school period.

"Most writers agree that drill should follow functional problems filled with socially useful content, rather than precede these problems. This may mean that we shall have to provide more opportunity for drill in later levels of maturity, and that provision in junior and senior high school should be made for maintenance activities.

"Not only is the present trend toward greater emphasis upon the social concepts involved in arithmetic, but also upon the importance of developing a sense of number meanings, and the ability to think in quantitative terms. This quantitative thinking seems to be increased with the introduction of a large number of comparatively simple but real-life problem solving situations. It would seem to be enhanced also by the introduction of many problems requiring that the pupil estimate results of a problem or that he attempt to approximate the numerical solution. It has always been my belief that more exercises in approximation or estimation would not only lead to a heightened sense of quantitative values but also to important habits of checking and accurate computation.

"I believe that it would be difficult to over-emphasize the importance of the right balance between skill in the swift and accurate computation of simple arithmetic combinations and the ability to solve socially useful problems involving quantitative thinking. It has been pointed out over and over again that these problems must present a personal challenge to

[5] To achieve any worthwhile results such a program should cover at least the work of both the ninth and tenth years. Editor.

the pupil, else they are not, in the strictest sense of the term, problems to him.

"Another line of development in recent writing and research on the teaching of arithmetic seems to point toward a greater emphasis upon generalization rather than too narrow stress on the specificity of arithmetic learning. A number of studies have indicated the possibility and actual fact of considerable transfer of training from the study of one arithmetic process to other related processes. Stress on the more generalized approach to arithmetic problems should also contribute toward the development of a sense of number values.

"From the standpoint of civic importance, no phase of arithmetic has such outstanding merit as the ability on the part of each individual to evaluate legislation, economic proposals, personal budgets, etc. in terms of their quantitative implications."

The junior high school people have never had an opportunity of working out this problem of what mathematics to teach in the ninth grade, because it was assumed that certain students needed four years of high school mathematics, and the only way to get this was to force algebra into the junior high school as a ninth grade subject. Many mathematicians are now desirous of breaking down the adamant walls between arithmetic, algebra, and geometry, and favor building a unified three year program of mathematics for the junior high school, using those sections and skills from arithmetic, algebra, trigonometry and geometry, which functional needs dictate as being desirable for the work undertaken. Projects could be built up around

1. The practical mathematics of any day in life—buying commodities and services, insurance, rates of interest, installment buying, budgeting, use of simple formulas and equations, use and interpretation of graphs, simple probability and chance, mensuration of simple geometric forms around us, etc.
2. Appreciational mathematics, its permeation of life in machines, engineering projects, form, figure, similarity, symmetry, art, architecture, design, etc.
3. Mathematics as a pattern of thought, simple inductive and deductive principles, importance of definitions and assumptions, understanding of authority and its use in thinking and self-discipline.
4. Emphasis upon quantitative thinking, etc.

These projects taught according to the maturation level of the child would dictate the skills and drills which would be taught in the ninth grade. The people who support this view are confident that upon this as a base could be built during three years of senior high school, a program of mathematics quite adequate for college entrance.

Although studies have shown rather definite gains in the lower elementary grades from the postponement of formal arithmetic, yet it is also quite well established that because of the lowering of average chronological age together with other causes, that students are not adequately equipped with the skills and their use in social situations at the end of the eighth grade. Therefore, it seems advisable that in the ninth grade students have more basic mathematics, greater facility and accuracy in the application of this basic material, and less abstract algebraic symbolism without direct and immediate application.

### Personnel of the Panel

Presiding: Earl Murray, Assistant Director of Curriculum, Santa Barbara Schools, Teacher of Mathematics, Santa Barbara High School, Santa Barbara, California.

Panel Members: Mr. H. Barret Patton—Washington Junior High School, Fresno, California; Mr. Vaden Mayers—San Mateo High School, San Mateo, California; Frank B. Lindsay—Assistant

Chief, Department of Secondary Education, State Department of Education, Sacramento, California; Miss Dorothy King—Frick Junior High School, Oakland, California; Miss Alice Wilson—Marina Junior High School, San Francisco, California; Mr. Robert Ritchie—Santa Barbara Junior High School, Santa Barbara, California; Dr. H. C. Christofferson—President of the National Council of Teachers of Mathematics, Professor of Mathematics in Miami University, Oxford, Ohio; Dr. D. Welty Lefever—Associate Professor of Education, University of Southern California, Los Angeles, California.


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*By* M. W. Keller, D. R. Shreve, and H. H. Remmers
*Purdue University, Lafayette, Indiana*

It is desirable that the college teacher know what numerical manipulations his students can do when they enter freshman mathematics courses so that he will be able to remedy the more glaring deficiencies as the course work proceeds, in order that the students may not be handicapped throughout the course, or even sometimes fail as a consequence of their disabilities. The high school and grade teachers are also interested in knowing what manipulative ability the students have retained so that they may increase the emphasis on those topics which students in general seem to forget, or have not mastered sufficiently well.

In an attempt to determine the knowledge and the ability in the use of mathematical techniques suppposedly at the command of beginning students in freshman courses in mathematics at Purdue University, a set of three diagnostic tests was constructed by the authors. These tests may be classified as technique tests since they only sought to determine the student's ability to understand and perform correctly fundamental manipulative operations. Consequently, no so-called reasoning or word problems were included. The first test included the fundamental operations on signed numbers and elementary percentage.

It is the purpose of this paper to discuss briefly the population tested and the test, and to present the factual information obtained from this first test on the ability of the students to perform the indicated signed number operations.

No attempt was made to choose the group tested. They were the students assigned to Drs. R. H. Downing, M. W. Keller, and D. R. Shreve by a regular nonselective process of enrollment during the first semester of the school year 1939–40. That the group was a fairly good random sample of the entering freshman class was verified by checking the placement test percentile scores. The students tested consisted of 204 who were enrolled in the School of Engineering first semester mathematics course and 75 who were enrolled in the School of Science college algebra course. This was approximately 20% of the total enrollment in each of these courses. The results obtained for the two groups were kept separately, since those enrolled in the School of Engineering are required to present more mathematics for entrance than the others. However, for this paper the results will be given for the entire group as a unit.

The test was composed of 32 pairs of problems. That is, each problem was matched with a second problem which involved the same operations, and in terms of the judgments of the investigators was of equal difficulty. That they were of approximately equal difficulty was verified from the actual results obtained on the test. The coefficient of correlation ("odds vs. evens") was 0.936. On the test the matched problems were so arranged that no pair was close together. Each type of problem included was so selected as to involve only those manipulative processes which occur in college algebra and trigonometry problems as determined from an analysis of the problems in the textbooks used at Purdue University. The test was mimeographed with a place provided by

each problem for the answer and necessary computations. It was given at the second meeting of each class without previous notice.

Since a student may know the manipulative processes involved and still make an error on one of the problems of a pair, we shall take as a measure of the students' inability (S.I.) to solve a certain type of problem the sum of those who were unable to do either problem, those who omitted both, and those who got one wrong and omitted the other one of the matched pair, divided by the total number of students. This should be a fairly accurate measure. A slight error may be introduced if a student makes a careless error on both problems. In problems as simple as those used this is not likely to occur frequently, and consequently the error introduced by this method of measuring will be relatively small.

It is also of interest to know just how accurate the students are in carrying out a manipulation which they know how to perform. As a measure of student accuracy (S.A.) we shall take the ratio of the total number of students who had both problems correct to the number who had both correct plus those who had the correct answer for one of the matched pair.

In order to get a complete view of the test and the resulting information, the accompanying table is given. This table includes the problem pairs which were used in this test. These problems have been grouped together by pairs for the purposes of this discussion. In addition the student inability (S.I.) and student accuracy (S.A.) as previously defined are given. The problem pairs are listed according to student inability.

From the table certain facts can readily be deduced. A few of the more important ones will be discussed. It is quite apparent from problem pair 32 that most of the students do not understand that division by zero leads to difficulty when one considers that any answer which indicated a recognition of an unusual situation was counted correct. The number of students who knew that division by zero was an excluded operation was negligible. Most responses which were counted correct gave the actually incorrect answer "infinity." This shows that if the students have been taught that "division by zero is impossible" if the fundamental operational postulates are to be consistent, it has never been sufficiently emphasized. This concept is of fundamental importance in mathematics. Had the students thoroughly understood the process of division as the inverse operation from multiplication, and had they reflected at all, they would have seen at once the inconsistency of their answers.

In problem pair 31 is displayed a second glaring deficiency, the inability to carry a division to a specified number of places and "round off" correctly. That is, in both problems, the fourth decimal was more than 5 so that it was necessary to increase the third decimal place by one in order to have the correct answer.

The problems in percentages present some rather interesting information. Thus it is seen from problem pair 4 that this type of manipulation is definitely well understood. If the students are required, however, to perform the same operation on a fraction as in problem pair 24 then there is a definite inability to handle it. A somewhat similar situation occurs with regard to problem pairs 11 and 27. If the percentage problems are considered as a group then it does not seem that the students understand the fundamental concepts any too well.

The accuracy in manipulation was far from satisfactory in every instance where any detailed computing was required. This could no doubt be materially improved by some intensive drill stressing accuracy and by demanding of students in all courses in elementary high school and university mathematics that they not only understand the process, but also that they do the manipulative work correctly. Though it is not possible to over-stress the understanding of the mathematical principles

involved in the solution of any problem, still the accuracy objective should never be sacrificed in attempting to attain an understanding of the principles.

| Problem Pair | Problems | S.I. | S.A. |
|---|---|---|---|
| 1. | $63\times\frac{8}{9}=$ ; $56\times\frac{9}{8}=$ | .01 | .89 |
| 2. | $3^3=$ ; $2^3=$ | .02 | .92 |
| 3. | $\frac{4}{15}-\frac{2}{9}=$ ; $\frac{5}{6}-\frac{3}{8}=$ | .04 | .81 |
| 4. | 17% of 93 = ; 13% of 97 = | .05 | .80 |
| 5. | Write $\frac{3}{4}$ as a decimal; Write $\frac{3}{5}$ as a decimal | .07 | .91 |
| 6. | $\sqrt{\frac{16}{81}}=$ ; $\sqrt{\frac{25}{49}}=$ | .07 | .85 |
| 7. | $\frac{3}{7}\div\frac{5}{8}=$ ; $\frac{4}{9}\div\frac{5}{7}=$ | .07 | .84 |
| 8. | $\frac{35.7}{.07}=$ ; $\frac{27.9}{.09}=$ | .07 | .82 |
| 9. | $41\times\frac{5}{7}=$ ; $31\times\frac{5}{9}=$ | .07 | .76 |
| 10. | $\frac{15}{91}\times 0=$ ; $\frac{21}{32}\times 0=$ | .08 | .91 |
| 11. | Convert $\frac{3}{4}$ to per cent; Convert $\frac{3}{5}$ to per cent | .08 | .90 |
| 12. | $(.2)^3=$ ; $(.3)^3=$ | .08 | .89 |
| 13. | $\frac{5}{39}+\frac{2}{13}=$ ; $\frac{5}{56}+\frac{3}{14}=$ | .08 | .84 |
| 14. | $97.6\times.0708=$ ; $79.6\times.0807=$ | .08 | .68 |
| 15. | $\frac{0}{43.7}=$ ; $\frac{0}{47.3}=$ | .09 | .90 |
| 16. | Write .007 as a fraction; Write .009 as a fraction | .09 | .87 |
| 17. | $\frac{7}{18}\div\frac{13}{18}=$ ; $\frac{8}{15}\div\frac{17}{15}=$ | .10 | .85 |
| 18. | $\frac{13}{23}-\frac{17}{23}=$ ; $\frac{15}{31}-\frac{21}{31}=$ | .13 | .84 |
| 19. | Simplify: $\sqrt{\frac{5}{9}}=$ ; Simplify: $\sqrt{\frac{3}{4}}=$ | .18 | .78 |
| 20. | $\frac{44}{35}\div\frac{33}{40}=$ ; $\frac{45}{22}\div\frac{36}{55}=$ | .18 | .62 |
| 21. | $\left(\frac{\sqrt{5}}{3}\right)^2=$ ; $\left(\frac{\sqrt{3}}{2}\right)^2=$ | .19 | .94 |
| 22. | What per cent of 50 is 9?; What per cent of 25 is 7? | .19 | .78 |
| 23. | $.84\div\frac{3}{7}=$ ; $.81\div\frac{3}{8}=$ | .19 | .64 |
| 24. | 7% of $\frac{7}{3}=$ ; 3% of $\frac{3}{7}=$ | .19 | .62 |
| 25. | $\frac{19}{11}/\frac{19}{8}=$ ; $\frac{17}{13}/\frac{17}{11}=$ | .25 | .88 |
| 26. | Write 2.7% as a decimal; Write 3.9% as a decimal | .27 | .93 |
| 27. | Write .007 as a per cent; Write .009 as a per cent | .28 | .90 |

| Problem Pair | Problems (continued) | S.I. | S.A. |
|---|---|---|---|
| 28. | (correct to 1 decimal) $.68 \div .0079 =$ ; $.78 \div .0089 =$ | .31 | .37 |
| 29. | Simplify: $\frac{3}{\sqrt{2}} =$ ; Simplify: $\frac{2}{\sqrt{3}} =$ | .37 | .88 |
| 30. | Write 2.7% as a fraction; Write 3.9% as a fraction | .37 | .88 |
| 31. | (correct to 3 decimals) $3.798 \div 8.7 =$ ; $3.897 \div 9.8 =$ | .67 | .48 |
| 32. | $7.9 \div 0 =$ ; $8.3 \div 0 =$ | .76 | .90 |

## The Doctrine of Postponement

AT THE conference of the Society for the Promotion of Engineering Education, in Berkeley, California, during the last week of June, the Council of the Society, acting for the Society, adopted the following preamble and resolution. The Executive Committee of the Council was instructed to transmit it to educational groups to be selected by the Committee, with power, such groups to be selected because of their position to influence the schools in behalf of the resolution. This resolution deserves consideration, for it comes from a group of men who do not teach mathematics, but who know the need of the subject.

"In various parts of the country there seems to be a movement to postpone and abbreviate the courses in mathematics given in the secondary schools. This movement apparently does not recognize the fact that these courses are essential prerequisites for the future training of scientific and engineering students, and that the university has not postponed and cannot postpone the mathematical or the scientific and engineering instruction in the university, if its graduates are to enter those professional fields. Moreover, at the present time, for our own defense as a nation, it is suicidal not to develop the most thorough kind of training for engineers.

"We wish to go on record as recommending that there be no postponement in the mathematical education in the secondary schools of those students who are to seek careers in science and engineering. In particular we feel that it is essential that a full four-year program of mathematics be available in the high schools for capable students, beginning with a year of college preparatory algebra in the ninth grade. We feel that this subject should not be postponed and also that thorough work in trigonometry and in solid geometry should be available.

"This resolution in no way implies that university preparatory courses be required of all students. But this group feels very strongly the importance of providing substantial courses in mathematics for those who need them in preparation for future work or for those who choose to elect them. We believe that to be effective these courses must begin with algebra at least as early as the ninth year, that is, the first of the last four years in the secondary schools."

# Shall We Defer the Teaching of Algebra to the Tenth Year?*

*By* HARRIET A. WELCH
*Lowell High School, San Francisco, California*

IN THE April number of the monthly bulletin published by the California State Department of Education, there appeared the following statement: "Replies from 324 public high school principals establish that more than half of these institutions have moved algebra from the ninth to the tenth grade. Plane geometry is an eleventh year subject in more than a third of these schools; in some, it is even postponed to the twelfth year."

This, of course, is not news to those teaching in high schools; but it seems that an association devoted to the study of mathematics might well take time for its consideration. What has brought about the change? Is it a change for the better?

1. A primary cause has no doubt been the failure of too many pupils in algebra and geometry, so that principals and parents are dissatisfied.

2. Many (including some teachers of mathematics) believe that lack of maturity may be partially responsible for the failures.

3. It is necessary to give some pupils further training in arithmetic since they have been promoted on account of chronological age without mastering arithmetic.

4. The trend is to try to have all high school courses closely related to real life situations and the feeling persists that former courses in algebra and geometry do not so function.

5. Perhaps, because some people, who are not prepared to teach elementary algebra, have too often been assigned classes in it, pupils have learned mechanical rules and not fundamental principles. The teachers who have later taught these pupils are dissatisfied with results. This is not universally true. Some of our pupils acquit themselves creditably when questions of algebraic nature arise in geometry.

One of the most difficult questions to answer is just how much the difference in the student's age affects his grasp of the subject. Professor W. D. Reeve has during the past year published three editorials in THE MATHEMATICS TEACHER on the deferring of algebra. In one, replying to an article by W. B. Brown, Director of Secondary Curriculum, Los Angeles, he says, "It is clear that the idea held by Mr. Brown that algebra is too hard for junior high pupils is shared by many other educators, particularly those who are not interested in the mathematics program. The notion that algebra and geometry be 'pushed up' if followed to any large degree really means pushing these subjects out altogether. It is important to point out that the solution of the problem is to have them begin the study earlier not later than the custom. Of course," he continues, "a great deal depends upon what is taught in algebra."

Let us consider the difficulty due to poor preparation in arithmetic. No one would contend that there are not many pupils who require remedial arithmetic. However, is it not possible that pupils improve in arithmetic while studying algebra?

In THE MATHEMATICS TEACHER, November 1939, a paper was published "Does a year's exposure to algebra improve a pupil's arithmetic ability?" An experiment at the High School of Commerce in New York City is described which seems to have been carefully conducted and which showed such an improvement in the case of 200 pupils tested. Many

* Paper presented before the Northern California Section of the Mathematical Association of America, January 27, 1940.

teachers of algebra have hoped that the pupils were learning some arithmetic along with algebra. If we consciously aim to do this, we may make better progress in this respect.

A year ago at the meeting of this association in San Francisco, we heard a summary of the preliminary report of the Joint Commission of the Mathematical Association of America and the National Council on the Place of Mathematics in Secondary Education. Following is a quotation from that report:

"Although the ninth year program in this plan is one in general, or composite mathematics, the chief constituent is algebra but it is algebra, according to the pattern of recent years and not the algebra of two or more decades ago."

In outlining the work of the tenth year, the writers of the report say that while demonstrative geometry will be the central theme, other fields stressed previously are not to be ignored. It will perhaps be recalled that the report provides for some algebra and intuitive geometry and even trigonometry in the seventh and eight grades.

The advocates of the movement to defer algebra feel that a course in general mathematics largely composed of arithmetic is more beneficial to the majority than algebra. At first thought one may agree. Certainly those deficient in arithmetic, if capable of learning it, should have more training in it. However, in the case of better students, cannot what is of real value be given either earlier or introduced into the algebra? Some years ago, some high schools replaced ninth year algebra by general mathematics. Two criticisms were made at that time. The stronger students as one boy put it, thought it was spread too thin and the weaker ones were bothered by the shifting from topic to topic in various fields. It was with a feeling of relief to most people that we returned to ninth year algebra. Personally I believe that we can do no better than follow the plan outlined in the preliminary report of the Joint Commission, that is, improve the course in algebra but retain it in the ninth year.

May I once more quote Professor Reeve? He says, "Evidence is beginning to accumulate that certain communities throughout the country that discarded ninth grade algebra from the course of study are putting it back again where it used to be. It is to be expected that when parents realize what their children are losing by having no training in mathematics, they will demand a hearing."

Teachers realize that there is a group which should not attempt algebra in the ninth year or ever for that matter. It is of the better students that we think when we object to deferring it. Possibly the greatest injustice is to those young students who welcome a challenging new subject and are able to grasp it. It is immaterial whether this is for college preparation or not. And incidentally, many find algebra easier than arithmetic.

Considering the later plans of students, some boys and girls who do not go to college want as complete a high school education as possible which in many cases includes mathematics, especially where boys are interested in navigation or aviation.

It is claimed by some that courses for college preparatory purposes should be postponed until as near the time of college entrance as possible. It is evident if algebra is to be postponed, prospective engineers will be handicapped but many ninth grade pupils do not know their future plans. Unless a pupil studies mathematics, he does not know whether he is interested in it or not. Professor Bacon of Stanford University pointed out in his talk before the National Council how college students in social science and biology needed to know more mathematics than the minimum two years in high school. Moreover, some law schools advise more than two years of high school mathematics. So, we might well say to the college preparatory group what you need is not deferred mathematics but more mathematics.

Last semester, at the University of California, 200 were enrolled in trigonometry, 335 in intermediate algebra, 755 in analytic geometry, a total of 1290. This, I am told, is a fair estimate of the number who take mathematics in their first semester at the university. Certainly, the 755 in analytic geometry would have been at some disadvantage if they had not taken a complete course in secondary mathematics.

Note also, 200 took trigonometry, 335 high school algebra, and 75 solid geometry in college. If they were continuing in mathematics or engineering (and 308 freshmen were enrolled in engineering) time needed for other courses was being used for high school work.

There are still many schools where algebra is taught in the ninth year and geometry in the tenth. Why not then teach arithmetic to those who need it without having the others mark time? Cannot we postpone algebra and geometry when necessary without holding back those ready for these subjects? There are in our high schools pupils capable of studying profitably more mathematics than is offered. Let us not take away the opportunities they now have.

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## Fiftieth Anniversary of the University of Chicago

MORE THAN 500, members of three mathematical organizations, the American Mathematical Society, the Mathematical Association of America, and the Algebra Conference, will attend meetings at the University of Chicago in 1941, held in connection with the celebration of the University's fiftieth anniversary.

The three mathematical groups are among thirty-one learned societies, more than 15,000 of whose members will gather at the University in its Anniversary Year, which begins in October, 1940, culminating in an academic festival in September, 1941.

Announcement at the University of the meetings was made by Frederic Woodward, vice-president emeritus and director of the Fiftieth Anniversary Celebration. The mathematicians will meet in the last week of August and the first week of September, 1941.

One of the youngest of the great American endowed universities, the University of Chicago reaches the fiftieth anniversary of its founding recognized as an eminent pioneer in education and research.

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THE LATE Ambassador Walter Hines Page was formerly editor of the *World's Work* and, like all editors, was obliged to refuse a great many stories. A lady once wrote him: "Sir: You sent back last week a story of mine. I know that you did not read the story, for as a test I had pasted together pages 18, 19, and 20, and the story came back with those pages still pasted; and so I know you are a fraud and reject stories before reading them."

Mr. Page wrote back: "Madame: At breakfast when I open an egg I do not have to eat the whole egg to discover it is bad."

# ◆ THE ART OF TEACHING ◆

## Teaching the Subtraction of Signed Numbers

*By* A. E. Katra
*University High School, University of Illinois, Urbana, Illinois*

Most textbook writers are agreed that the best way to introduce the addition of signed numbers is by the means of a horizontal or vertical scaled line. When the vertical line is used the direction upward is positive while downward is negative, and addition is carried out by placing a marker on the point of the line corresponding to one of the addends and then counting off as many units upwards or downwards as suggested by the other addend. The sum of the two addends is that number on the scaled line where the marker has stopped after the operation.

However, for the subtraction of two numbers these authors advocate a method in which the difference is determined by reading the number of units and the direction taken on the scaled line in proceeding from the subtrahend to the minuend. Logically and pedagogically this manner of subtraction is not as sound as the following effective method.

The scaled line is a device designed to give some concrete basis to the secondary school student for his introductory work with signed numbers. If this device is used in one way for reading off the answers to addition and later in an entirely new way for reading off the results of subtraction, confusion is bound to result. It is much better to treat subtraction as the "opposite" of addition so that the results in both operations are determined by the number on the scaled line where the marker has stopped at the conclusion of the operation.

To subtract, therefore, place a marker on that point of the scaled line which corresponds to the minuend; the magnitude of the subtrahend indicates the number of units to be counted off, the direction of the counting (opposite to that taken if this were an addition problem) being determined by the sign of the subtrahend. The answer is then read off in exactly the same way as in addition.

For example: In the exercise $(+7)-(-5)=?$ the marker is placed on the point $(+7)$ of the scaled line. Now, if the $(-5)$ was to be added to the $(+7)$ we would count off 5 units downwards, but since this is a subtraction problem we must expect a different result from that obtained in addition, hence we have no choice but to proceed upwards 5 units. The marker now rests on $(+12)$.

Another example: $(-3)-(+8)=?$ Place marker on the point of the scaled line corresponding to $(-3)$. If we were adding $(+8)$ we would operate upwards 8 units; for subtraction, therefore, we must go down 8 units. The marker will now rest on $(-11)$, the difference.

This method of subtraction has two outstanding merits. Once the student has learned how to use the scaled line for addition in the approved manner, subtraction follows easily as the logical "opposite" of addition with no chance for confusion in the reading of answers. More important, however, this new method provides a concrete illustration for the double interpretation of the $(+)$ and $(-)$ signs as symbols of operation or opposition. The fact that $(+7)+(-5)$ will give the same results as $(+7)-(+5)$ can be demonstrated and some reasonable explanation can be given for the identical answers.

In conclusion it should be emphasized again that the use of the scaled line does not prove anything; it is merely a device.

# ◆ IN OTHER PERIODICALS ◆

*By* NATHAN LAZAR
*The Bronx High School of Science, New York City*

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# The National Council of Teachers of Mathematics Official Notice

As Secretary of the National Council of Teachers of Mathematics, I offlcially announce the annual election of certain officers of the National Council, said election to take place at Atlantic City, New Jersey, February 22, 1941. Article III Section 7 of the By-laws states: "At least two months before the date of the annual meeting, all members shall be given the opportunity, through announcement in the official journal, to suggest by mail for the guidance of the directors a candidate for each elective office for the ensuing year. At least one month before the annual meeting the secretary of the board of directors shall send to each member an official ballot giving the names of two candidates for each office to be filled. These candidates shall be selected by a nominating committee of the board of which the secretary shall be chairman. The election shall be by mail or in person and shall close on the date of the annual meeting."

At the Atlantic City Meeting, February 26, 1938, of the National Council the nominating committee consisting of the two most recent ex-presidents and the secretary as chairman (for this year: Martha Hildebrandt, H. C. Christofferson, and Edwin W. Schreiber), was instructed to prepare an official ballot suggesting two eligible candidates for each elective office and reserve a blank space for a third prospective candidate which may be written in by the voter. The officers to be elected at the 1941 Atlantic City Meeting are: Second Vice President, 1941–1942; and three Directors, 1941–1943. Since there is to be no primary ballot, affiliated organizations, local clubs, or individual members who wish to present names for the official ballot should do so by writing the secretary immediately. The official ballot will be sent to members through the mail the first week in January.

The periods of service of the officers of the National Council, from its organization in 1920 to the present time, are printed on page 331.

Edwin W. Schreiber, *Secretary*

# The National Council of Teachers of Mathematics

Organized 1920—Incorporated 1928

*Periods of Service of the Officers of the National Council*

*Honorary Presidents*

*H. E. Slaught, Chicago, Ill, 1936–1937

*Presidents*

1. C. M. Austin, Oak Park, Ill., 1920
2. J. H. Minnich, Philadelphia, Pa. 1921–1923
3. Raleigh Schorling, Ann Arbor, Mich., 1924–1925
4. Marie Gugle, Columbus, Ohio, 1926–1927
5. Harry C. Barber, Exeter, N. H., 1928–1929
6. John P. Everett, Kalamazoo, Mich., 1930–1931
7. William Betz, Rochester, N. Y., 1932–1933
8. J. O. Hassler, Norman, Okla., 1934–1935
9. Martha Hildebrandt, Maywood, Ill., 1936–1937
10. H. C. Christofferson, Oxford, Ohio, 1938–1939
11. Mary A. Potter, Racine, Wis. 1940–1941

*Vice-Presidents*

1. H. O. Rugg, New York City, 1920
2. E. H. Taylor, Charleston, Ill., 1921
3. Eula Weeks, St. Louis, Mo., 1922
4. Mable Sykes, Chicago, Ill., 1923
5. Florence Bixby, Milwaukee, Wis., 1924
6. Winnie Daley, New Orleans, La., 1925
7. W. W. Hart, Madison, Wis., 1926
8. C. M. Austin, Oak Park, Ill., 1927–1928
9. Mary S. Sabin, Denver, Colo., 1928–1929
10. Hallie S. Poole, Buffalo, N. Y., 1929–1930
11. W. S. Schlauch, New York City, 1930–1931
12. Martha Hildebrandt, Maywood, Ill., 1931–1932
13. May A. Potter, Racine Wis., 1932–1933
14. Ralph Beatley, Cambridge, Mass., 1933–1934
15. Allen R. Congdon, Lincoln, Neb., 1934–1935
16. Florence Brooks Miller, Shaker Heights, Ohio, 1935–1936
17. Mary Kelly, Wichita, Kansas, 1936–1937
18. John T. Johnson, Chicago, Ill., 1937–1938
19. Ruth Lane, Iowa City, Iowa, 1938–1939
20. E. R. Breslich, Chicago, 1939–1940
21. F. L. Wren, Nashville, Tenn. 1940–1941

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# Seventh December Meeting of the National Council of Teachers of Mathematics

To be held jointly with the Mathematical Association of America and the American Mathematical Society at Baton Rouge, Louisiana, December 30, 1940–January 1, 1941.

General Theme: The Relation Between Enriched Mathematical Experience and Enriched Community Experience.

The sessions of the Council will begin Monday morning December 30th and continue through Wednesday morning January 1st. The regular sessions of the Mathematical Association will be held on Thursday morning and afternoon January 2nd. A joint session of the Mathematical Association and the National Council is scheduled for Wednesday morning. A joint dinner of the three mathematical organizations is planned for Wednesday evening.

An excursion to St. Francisville and its environs to visit some of the old colonial residences is planned for Tuesday afternoon. Other alternative excursions may be planned later.

### Monday, December 30

1. General Meeting
   10:00 a.m. Himes Hall, Room 104
   Presiding—Ruth Lane, University of Iowa, Iowa City, Iowa
   1. What Should Enriched Community Experience Mean to the Secondary School Pupil?
      Dean Clarence A. Ives, Louisiana State University, Baton Rouge, La.
   2. What Should Enriched Mathematical Experience Mean to the Secondary School Pupil?
      W. W. Hart, Kenilworth, Ill.
   3. How May Teachers of Secondary School Mathematics Emphasize the Relationship Between Enriched Mathematical Experience and Enriched Community Experience?
      Etoile Wright, City Schools, Tulsa, Okla.
2. Senior High School
   1:30 p.m. Himes Hall, Room 204
   Presiding—A. R. Congdon, University of Nebraska, Lincoln, Neb.
   1. Word Problems in Algebra
      Elizabeth Dice, North Dallas High High School, Dallas, Tex.
   2. The Social Studies Curriculum and Mathematics
      Dorothy McCoy, Belhaven College, Jackson, Miss.
   3. What Price Enrichment?
      J. O. Hassler, University of Oklahoma, Norman, Okla.
3. Junior High School
   1:30 p.m. Himes Hall, Room 104
   Presiding—H. C. Christofferson, Miami University, Oxford, Ohio
   1. Some Perplexing Questions Relating to Junior High School Mathematics.
      R. Schorling, University of Michigan, Ann Arbor, Mich.
   2. Do We Practice What We Preach?
      Veryl Schult, Supervisor of Mathematics, Washington, D.C.
   3. The Logical Result of Enriched Pupil Experience.
      Nelson Tull, East Side Junior High School, Little Rock, Ark.
4. Teacher Training
   1:30 p.m. Himes Hall, Room 201
   Presiding—G. H. Jamison, State Teachers College, Kirksville, Mo.
   1. The Training of Southern Association Teachers of Secondary Mathematics.
      H. T. Karnes, Louisiana State University, Baton Rouge, La.
   2. The Role of Mathematics in Our Present Civilization.
      H. E. Buchanan, Tulane University, New Orleans, La.
5. Mathematical Movies
   3:00 p.m. Nicholson Hall, Room 104
   Presiding—Ruth Stokes, Winthrop College, Rock Hill, S. C.
   3:00—Circles
   3:15—Know your Money
   3:30—Geometry in Action
   3:45—Rate of Change
   3:55—The Equation $\ddot{X}+X=0$
   4:05—A Hypocyclic Motion
   4:10—Individual Differences in Arithmetic
   4:30—Parabola
   4:45—The Generation of Involute Gear Teeth
   4:55—Modes and Motors
   5:10—Precisely So
6. Multisensory Aids
   10:30 a.m.
   Presiding—E. H. C. Hildebrandt, State Teachers College, Upper Montclair, N. J.
   1. Fourth Dimensional Models in Secondary School Mathematics
      Harriet Herbert, Edmunds High School, Sumter, S. C.

2. Geometry via the Laboratory
   Robert C. Yates, Louisiana State University
3. Visual Aids—Student Made
   Kate Bell, Lewis and Clark High School, Spokane, Wash.

7. General Meeting
   8:00 p.m. Nicholson Hall, Room 104
   Presiding—J. O. Hassler, University of Oklahoma, Norman, Okla.
   1. Address of Welcome
      S. T. Sanders, Head of Department of Mathematics, Louisiana State University, Baton Rouge, La.
   2. Response of President of National Council of Teachers of Mathematics.
      Mary Potter, Supervisor of Mathematics, Racine, Wis.
   3. Address
      W. B. Carver, Cornell University, Ithaca, N.Y.

TUESDAY, DECEMBER 31

8. Senior High School
   8:30 a.m. Himes Hall, Room 204
   Presiding—W. D. Reeve, Teachers College, Columbia University, New York, N.Y.
   1. Use of Applications for Instructional Purposes.
      E. G. Olds, Carnegie Institute of Technology, Pittsburgh, Pa.
   2. Panel Discussion

   Members of Panel—E. G. Olds, Chairman
   Kate Bell, Lewis and Clark High School, Spokane, Wash.
   W. J. Bordelon, University High School, University, La.
   Mrs. H. L. Garrett and Several Pupils, Instrouma High School, University, La.
   H. S. Kaltenborn, Louisiana Polytechnic Institute, Ruston, La.
   Thirza Mossmann, Kansas State College, Manhattan, Kansas.
   Frank A. Rickey, Louisiana State University, Baton Rouge, La.

9. Teacher Training
   8:30 a.m. Himes Hall, Room 201
   Presiding—L. H. Whitcraft, Ball State Teachers College, Muncie, Ind.
   1. Assistance Needed by Mathematics Teachers in Service
      Jessie May Hoag, Jennings High School, Jennings, La.
   2. The Place of the Supervisor in Training Mathematics Teachers in Service.
      H. G. Ayre, Western Illinois State Teachers College, Macomb, Ill.
   3. Desirable Training for Supervisors of Mathematics.
      R. Schorling, University of Michigan, Ann Arbor, Mich.

10. Arithmetic
    8:30 a.m. Himes Hall, Room 104
    Presiding—C. C. Richtmeyer, Central State Teachers College, Mt. Pleasant, Mich.
    1. Useful Mathematics that Children Enjoy.
       H. C. Christofferson, Miami University, Oxford, Ohio.
    2. A Meaningful Way of Teaching Division of Fractions.
       Thelma Tew, Florida State College for Women, Tallahassee, Fla.
    3. Making Arithmetic of Greatest Value to the Community.
       R. L. O'Quinn, Louisiana State University, Baton Rouge, La.

11. Visual Aids
    10:30 a.m. Nicholson Hall, Room 104
    To be announced.
    Tuesday Afternoon is open for excursions to points of interest.

12. Southern University Meeting
    7:00 p.m. Southern University, Room 114 Science-Agriculture Building.
    Presiding—S. T. Sanders, Louisiana State University, Baton Rouge, La.
    1. The National Council of Teachers of Mathematics; Its Purpose and Program.
       Mary Potter, Council President and Supervisor of Schools, Racine, Wis.
    2. How Council Publications Serve Teachers of Mathematics.
       W. D. Reeve, Publications Editor, Teachers College, Columbia University, New York, N.Y.
    3. How the National Council of Teachers of Mathematics May Serve Negro Teachers and How They May Serve the Council.
       Mrs. Ethel Harris Grubbs, Washington, D.C.

WEDNESDAY, JANUARY 1

13. Arithmetic
    8:30 a.m. Himes Hall, Room 104
    Presiding—Edith Woolsey, Minneapolis, Minn.
    1. Some Experimental Evidence on Mastery by the Meaning Method.
       C. Newton Stokes, Temple University, Philadelphia, Pa.
    2. The National Council Committee Report.
       R. Schorling, University of Michigan, Ann Arbor, Mich.
    3. How Instruction in Arithmetic Might Be Improved.
       J. O. Pettiss, Louisiana State University, Baton Rouge, La.

14. Joint Session of Council and Association
8:30 a.m. Nicholson Hall, Room 104
To be announced

Afternoon—Sugar Bowl Football Game, 1:15 p.m.

15. Joint Banquet with Association and Society
7:30 p.m.

Provision for registration has been made for the Association, Council, and Society in Evangeline Hall. There will be a place in Himes Hall also. Himes Hall, Room 110 has been provided as a conference and committee room. Anyone who might wish to schedule a committee meeting or a group conference should write to Dr. P. C. Scott, Louisiana State University.

Rooms are available in the dormitories of Louisiana State University at a rate of $1.50 per day for single rooms and $1.00, each per day for two or more in a room. In the Stadium Dormitory (for men) rooms are available for $1.00 for single rooms and 75 cents per day for two or more in a room. Meals will be served at reasonable rates at Highland Hall, the Faculty Club, and the Field House Restaurant.

Rooms may also be had at the Baton Rouge hotels whose rates are as follows: Hotel Heidelberg, single $2.50–$4.00, double $3.50–$5.00, all rooms having private bath. Hotel King and Istrouma Hotel, single $1.50–$3.00, double $2.50–$5.00. Hotel Louisian, single $1.50–$2.00, double $2.25–$3.50. In the King, Istrouma and Louisian single rooms at $2.00 and up and double rooms at $3.00 and up have private baths. Hotel reservations should be made early with the managers since the meeting occurs during their winter season and at the time of the Sugar Bowl football game.

Members are advised to consult their local agents concerning special winter tourist rates on the railroads.

Tickets for the Sugar Bowl game will be on general sale after November 1st. The sale of prices is as follows:

Box seats $5.50.

Reserved seats between 25 yd. lines $4.50.

All other seats in east and west stands, $3.50.

All seats in North and south stands, $2.75.

Negro section, $2.75.

All prices include tax. All seats are reserved. Requests for tickets should be made direct to Maison Blanch Co., Corner Canal and Daulphin Sts., New Orleans, La. Checks should be made payable to the Maison Blanche Co.

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## The Problem

Another hard problem in 'rithmetic?
What is it that's bothering now?
Come over here by the window, dear,
And Daddy will show you how.

"Ah! . . . Jane had twice as many as John;
We're told how many Jane had;
To find out the number John received
What must we do, honey?"
—"Add?"

"No, little girl; we'll try it once more;
This time think hard, Alice; try!
John had half as many as Jane;
What shall we do?"
—"Multiply?"

"Come! Come! don't watch the children out there!
That isn't the way to act!
Jane had ten, twice as many as John;
Can't you see what we do?"
—"Subtract?"

"You say 'add,' 'multiply,' and then 'subtract';
That isn't the way to decide
How examples are done; you must think them out;
Come! try once again."
—"Divide?"

"That's right; you divide; and what do you get?"
—"Five?"—"Yes; see what it's all about?
Is there something you'd like to ask me, dear?"
—"Daddy, may I go out?"

F.W.R.

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